Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 19, 2002*
Accepted: *May 5, 2002*

An exact holographic RG flow between 2d conformal fixed points

Marcus Berg

Department of Physics, University of Rome, Tor Vergata
Via della Ricerca Scientifica, 00133 Rome, Italy
E-mail: berg@roma2.infn.it

Henning Samtleben

Spinoza Insituut, Universiteit Utrecht
Leuvenlaan 4, Postbus 80.195, 3508 TD Utrecht, The Netherlands
E-mail: H.Samtleben@phys.uu.nl

Abstract: We describe a supersymmetric RG flow between conformal fixed points of a two-dimensional quantum field theory as an analytic domain wall solution of the three-dimensional $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauged supergravity. Its ultraviolet fixed point is an $N = (4, 4)$ superconformal field theory related, through the double D1-D5 system, to theories modeling the statistical mechanics of black holes. The flow is driven by a relevant operator of conformal dimension $\Delta = 3/2$ which breaks conformal symmetry, breaks supersymmetry down to $N = (1, 1)$, and sends the theory to an infrared conformal fixed point with central charge $c_{\mathrm{IR}} = c_{\mathrm{UV}}/2$.
Using the supergravity description, we compute counterterms, one-point functions and fluctuation equations for inert scalars and vector fields, providing the complete framework to compute two-point correlation functions of the corresponding operators throughout the flow in the two-dimensional quantum field theory. This produces a toy model for flows of $N = 4$ super Yang-Mills theory in 3+1 dimensions, where conformal-to-conformal flows have resisted analytical solution.

Keywords: AdS-CFT and dS-CFT Correspondence, Supergravity Models.

Contents

1. Introduction

In the last few years, the interest in the connection between gauge theories and string theory has been re-ignited by the AdS/CFT correspondence [1, 2, 3]. Extending the success of the correspondence to theories that flow to some conformal field theory at a fixed point is of great interest; this is accomplished by holographic renormalization group methods [4]–[8]. Using this extension of the correspondence, we describe a supersymmetric renormalization group (RG) flow between conformal fixed points of a two-dimensional quantum field theory. To the best of our knowledge, this is the first example of a holographic flow between conformal fixed points which is exact, i.e. an explicit solution for the bulk spacetime. The boundary theory is a large (or "double") $N = (4, 4)$ superconformal field theory [9, 10, 11], in which the flow is driven by a relevant operator of conformal dimension $\Delta = 3/2$, which we interpret as a mass term for chiral superfields, in close analogy with the flow of [12]. This $\Delta = 3/2$ operator breaks conformal symmetry, and it breaks supersymmetry to $N = (1, 1)$. Our general interest in these theories is twofold: first, this type of CFT is known to supply quantitative understanding of black hole quantum mechanics. Second, our flow can serve as a toy model for flows of physically relevant theories, a prominent example being flows of $N = 1$ super Yang-Mills (SYM) theory in $3+1$ dimensions. In the rest of the introduction, we expand on these two motivations and give an overview of the new results in this paper.

The above-mentioned $N = (4, 4)$ conformal field theory is related to theories modeling the statistical mechanics of black holes. An important example is the D1-D5 system on a torus T^4, which has a complementary supergravity description as a five-dimensional black hole with three charges. In [13], the $\mathrm{U}(1)^4$ symmetry of translations in the 4-torus was seen as a limit of an affine $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry where the ratio $\alpha = k^+/k^-$ of SU(2) levels becomes small. This larger symmetry was used to compute the density of states in the D1-D5 system on T^4 despite the vanishing of the elliptic genus in this case. Now, the affine $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is part of the large $N = 4$ superconformal algebra $\mathcal{A}_\gamma$ [14].[1] (As a reminder, the small $N = 4$ algebra only has one affine SU(2)).

In terms of branes, these large symmetries correspond to a "double" D1-D5 system [9, 11], from which the standard D1-D5 system is recovered in the limit $\alpha \to 0$ where the charges of one of the systems are much greater than the charges of the other. Close to this limit, the double D1-D5 system is an interesting (and rather puzzling) ten-dimensional deformation of the physically relevant D1-D5 system, even though its supergravity description might not be a deformation of a five-dimensional black hole in a five-dimensional sense. It remains somewhat mysterious; as emphasized in [11], strings stretching between the two D5-branes induce nonlocal couplings between the worldvolume theories.

At low energy, the double D1-D5 system yields an $AdS_3 \times S^3 \times S^3 \times S^1$ geometry admitting 16 Killing spinors, i.e. which is half maximally supersymmetric. The isometries of this geometry form two copies of the supergroup $D^1(2, 1; \alpha)$, where the ratio of brane charges α now coincides with the ratio of the two sphere radii. This supergroup shares the bosonic subgroup $\mathrm{SU}(2)^2 \times \mathrm{SU}(1, 1)$ with $\mathcal{A}_\gamma$. In this paper, we mostly concentrate on the case of $\alpha = 1$, when the isometry is two copies of $D^1(2, 1; 1) = \mathrm{OSp}(4|2)$.

From a black hole point of view, it is interesting to try to understand what remains of the relation to the black hole picture when less supersymmetry is present and conformal symmetry of the worldvolume theory is broken. Adding a Lorentz-invariant relevant operator to the lagrangian accomplishes this; in particular for operators that completely break supersymmetry, one could imagine an RG flow that describes temperature effects in the black hole. At the moment, we have nothing more to add about this — of course, without supersymmetry it is not clear how to control corrections in the transition to the supergravity regime.

A general reason to study holographic RG flows in two dimensions is simply to provide toy models for flows of strongly coupled gauge theories in higher dimensions, among which $N = 1$ SYM is the example of greatest phenomenological interest. In two dimensions, one can study features of the correspondence that are under much better control than in four dimensions. For instance, much information is encapsulated in the central charge c. The Zamolodchikov c-theorem [15] states that c is smaller at an infrared fixed point, i.e. that degrees of freedom associated to massive fields become unimportant at distances longer than the Compton wavelength of those massive excitations. Instead of holographically proving the c-theorem as in four dimensions, we have the luxury of a well-established result on the CFT side. Indeed, all fixed points we find satisfy the Zamolodchikov c-theorem.

[1] We use the standard notation k^+, k^-, for the levels of the two affine SU(2) factors, while γ is given by $\gamma = k^+/(k^+ + k^-) = \alpha/(1 + \alpha)$. The central charge of $\mathcal{A}_\gamma$ is $c = 6k^+k^-/(k^+ + k^-)$.

Our flow can be viewed as a two-dimensional analogue of the supersymmetric Freedman-Gubser-Pilch-Warner (FGPW) flow between $N = 4$ SYM and an $N = 1$ superconformal field theory in 3+1 dimensions, obtained by giving mass to one of the three chiral superfields [12]. (In contrast, the well-studied $N = 1^*$ theory flow [16, 17] corresponds to the three chiral superfields receiving equal mass.) In four dimensions, the spacetime holographically dual to this flow could only be described numerically. This is a great drawback as one would need an exact flow to be able to compute correlation functions. While there is no obvious reason why the flow equations themselves (which are always one-dimensional) should be simpler in three-dimensional spacetime than in five, we find an analytic solution to the three-dimensional Killing spinor conditions.

The tool of holographic renormalization is currently gauged supergravity. In the FGPW flow of $N = 4$ SYM in 3+1 dimensions, the Günaydin-Romans-Warner five-dimensional gauged supergravity, with an $E_{6(6)}/\,\mathrm{USp}(8)$ scalar manifold [18], provided the proper setting. Although the full nonlinear dimensional reduction of ten-dimensional supergravity on $AdS_5 \times S^5$ is not known, the aforementioned five-dimensional gauged supergravity has been argued to be a consistent truncation describing states of lowest mass in this reduction. The analogous framework for the holographic dual of a 1+1 field theory would be found among three-dimensional gauged supergravities; these theories were constructed in [19, 20]. In these papers, consistency conditions were condensed to a group theory condition from which a menu of allowed gauge groups could be compiled. Returning to the case of half-maximal supersymmetry, the gauge groups are particular subgroups [20] of the isometries of the scalar manifold $\mathrm{SO}(8,n)/(\mathrm{SO}(8)\times\mathrm{SO}(n))$. Since we are interested in compactifications where the internal geometry includes an $S^3 \times S^3$, the theory relevant for our analysis has local symmetry $\mathrm{SO}(4) \times \mathrm{SO}(4)$ with two independent coupling constants, corresponding to the isometry groups and radii of the two three-spheres, respectively. As we shall discuss in more detail below, this theory indeed reproduces the spectrum of lowest mass states found in the reduction of supergravity on $AdS_3 \times S^3 \times S^3 \times S^1$.

We should also emphasize that the undeformed boundary theory has some mysterious properties that we do not attempt to clarify in this paper. Unlike in the case of $N = 2$ and small $N = 4$ theories, the BPS relation between conformal dimension and charge in the large $N = 4$ theory is nonlinear, see equation (3.15). As pointed out in [11], the nonlinear contribution is subleading in $1/N$, hence the BPS mass formula receives string loop corrections. In supergravity, this nonlinear part seems invisible (contrast (2.11) and (3.15)), which makes it difficult to establish a precise correspondence including multiparticle states for general α. Nevertheless, in [10], a correspondence was proposed for rational values of the ratio α.

The paper is organized as follows. In section 2 we review the three-dimensional $N=8$ gauged supergravity with gauge group $\mathrm{SO}(4) \times \mathrm{SO}(4)$, describing the lowest mass states in the reduction of supergravity on $AdS_3 \times S^3 \times S^3 \times S^1$. We study the scalar potential in certain truncations and find several stable extremal points. In particular, we uncover two extrema which preserve $N = (1, 1)$ supersymmetry. The spectrum of physical fields around these extrema is given in section 2.3 and the appendix. In section 3 we present an analytic kink solution for the metric and the scalar fields which interpolates between the central maximum of the potential and one of the supersymmetric extrema. Interpreted as a holographic renormalization group flow, this solution flows between conformal fixed points with $c_{\rm IR}/c_{\rm UV} = 1/2$. We moreover find that all the operators in the IR theory have rational conformal dimension.

Section 4 contains the computation of counterterms for inert scalars and the 1-point functions of their CFT duals. In section 5 we derive the linearized fluctuation equations for inert scalars and vector fields around our flow solution. We show that they may all be reduced to two second-order "universal" differential equations. Together with the 1-point functions, the properly normalized solutions to these equations encode the entire information about 2-point correlation functions of the dual operators in the boundary theory. The appendix contains an explicit parametrization of the scalar potential of the gauged supergravity, and a collection of stable extrema together with their spectra.

2. $D = 3$, $N = 8$ supergravity with local $\mathrm{SO}(4) \times \mathrm{SO}(4)$

2.1 Lagrangian and local maximum

As discussed in the introduction, we are interested in compactifications where the internal geometry includes an $S^3 \times S^3$, leading to an $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge symmetry in the three-dimensional effective theory, with 16 real supercharges. This theory is the three-dimensional $N = 8$ gauged supergravity with local $\mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry. Its matter sector consists of n multiplets each containing 8 scalars and 8 fermions, whereas graviton, gravitini, and the 12 vector fields are non-propagating in three dimensions, see (2.3) and the subsequent discussion. An important difference to the maximally supersymmetric case is that from a three-dimensional point of view, we may turn on any number n of matter multiplets in the classical supergravity, although anomaly cancellation can constrain the value of n in the quantum theory. (In a three-dimensional bulk, there is no chiral anomaly, so it would have to come from the boundaries.) The $8n$ scalars parametrize the coset manifold $\mathrm{SO}(8,n)/(\mathrm{SO}(8) \times \mathrm{SO}(n))$.

The lagrangian of this theory is given by [20]

$$\mathcal{L} = -\frac{1}{4}\sqrt{G}R + \mathcal{L}_{CS} + \frac{1}{4}\sqrt{G}G^{\mu\nu}\mathcal{P}_\mu^{Ir}\mathcal{P}_\nu^{Ir} - g^2\sqrt{G}V + \mathcal{L}_F\,, \tag{2.1}$$

where $\mathcal{L}_F$ contains the fermionic terms, explicitly given in (2.24) below, and $G_{\mu\nu}$ is the bulk metric. We use signature $(+--)$. Indices $I, J, \dots$ and indices $r, s, \dots$ denote the vector representations of $\mathrm{SO}(8)$ and $\mathrm{SO}(n)$, respectively. The 12 vector fields transform in the adjoint representation of the gauge group

$$\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^- \subset \mathrm{SO}(8) \subset \mathrm{SO}(8,n)\,, \tag{2.2}$$

where we use $+$ and $-$ superscripts to distinguish the two three-spheres. The vector fields are collectively denoted by ${B_\mu}^{IJ} = {B_\mu}^{[IJ]}$, for $I, J \in \{1,2,3,4\}$ or $I, J \in \{5,6,7,8\}$, respectively, corresponding to the two factors in (2.2). In contrast to higher-dimensional gauged supergravities, the dynamics of the vector fields is governed by a Chern-Simons term

$$\mathcal{L}_{CS} = -\frac{1}{4}\epsilon^{\mu\nu\rho} g\Theta_{IJ,KL} B_\mu{}^{IJ} \left(\partial_\nu B_\rho{}^{KL} + \frac{8}{3} g \eta^{KM} \Theta_{MN,PQ} B_\nu{}^{PQ} B_\rho{}^{NL} \right), \tag{2.3}$$

indicating that this dynamics is pure gauge, i.e. the vector fields do not carry physical degrees of freedom.[2] Here, the parameter g denotes the gauge coupling constant, and the tensor $\Theta_{IJ,KL}$ describes the embedding of the gauge group into SO(8):[3]

$$\Theta_{IJ,KL} = \begin{cases} \alpha\epsilon_{IJKL} & \text{for } I,J,K,L \in \{1,2,3,4\} \\ \epsilon_{IJKL} & \text{for } I,J,K,L \in \{5,6,7,8\} \\ 0 & \text{otherwise} \end{cases} . \tag{2.4}$$

The free parameter α describes the ratio of coupling constants of the two SO(4) factors (2.2), alias the ratio of radii of the two three-spheres or the ratio of charges of the two D1-D5 systems, cf. the discussion in the introduction. The scalar sector in (2.1) is parametrized by SO$(8,n)$ matrices $\mathcal{S}$ which define the currents $\mathcal{P}_\mu^{Ir}$ according to

$$\mathcal{S}^{-1}\mathcal{D}_\mu \mathcal{S} = \mathcal{S}^{-1}\left(\partial_\mu + g\Theta_{IJ,KL} B_\mu{}^{IJ} X^{KL}\right)\mathcal{S} = \frac{1}{2}\mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir}, \tag{2.5}$$

where X^{IJ}, X^{rs} denote the compact generators of SO$(8,n)$, and Y^{Ir} the noncompact ones. The scalar potential is given by

$$V = -\frac{1}{4}\left(A_1^{AB} A_1^{AB} - \frac{1}{2} A_2^{A\dot{A}r} A_2^{A\dot{A}r} \right), \tag{2.6}$$

in terms of the SO(8) tensors A_1, A_2

$$\begin{aligned} A_{1AB} &= -\frac{1}{48}\Gamma^{IJKL}_{AB} \mathcal{V}^{MN}{}_{IJ} \mathcal{V}^{PQ}{}_{KL} \Theta_{MN,PQ}, \\ A_{2A\dot{A}r} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}} \mathcal{V}^{MN}{}_{IJ} \mathcal{V}^{PQ}{}_{Kr} \Theta_{MN,PQ}, \end{aligned}$$

with the scalar matrix $\mathcal{V}$ obtained from expanding

$$\mathcal{S}^{-1} X^{IJ} \mathcal{S} = \frac{1}{2}\mathcal{V}^{IJ}{}_{MN} X^{MN} + \frac{1}{2}\mathcal{V}^{IJ}{}_{rs} X^{rs} + \mathcal{V}^{IJ}{}_{Kr} Y^{Kr}.$$

The lagrangian (2.1) has a local SO(8) $\times$ SO(n) symmetry (corresponding to the redundancies of the coset structure SO$(8,n)/(\mathrm{SO}(8)\times\mathrm{SO}(n))$), which acts by right multiplication

[2] Specifically, one may view the vector fields as nonlocal functions of the scalar fields, entirely defined by the first order duality equations induced by (2.3) up to local gauge freedom. Equivalently, one may fix this gauge by eliminating some of the scalar fields, whereby the vector-scalar duality equations become massive self-duality equations for the vector fields. This is illustrated in the linearized analysis in section 5.

[3] Our conventions for the ϵ-symbols are $\epsilon_{1234} = \epsilon_{5678} = \epsilon_{12345678} = 1$, while we use a representation of SO(8) Γ-matrices in which $\Gamma^{[4]}$ is selfdual according to $\Gamma^{IJKL} = \epsilon_{IJKLMNPQ}\Gamma^{MNPQ}$.

JHEP05(2002)006

on $\mathcal{S}$. The local $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge symmetry acts by left multiplication on $\mathcal{S}$. In addition, (2.1) possesses a remaining global $\mathrm{SO}(n)$ symmetry which likewise acts by left multiplication on $\mathcal{S}$, rotating the n matter multiplets.

The potential (2.6) has a local maximum at $\mathcal{S} = I_{8n}$, which yields an $N = (4,4)$ supersymmetric AdS_3 solution of (2.1). The value of the potential at this point is

$$V_0 = V|_{\mathcal{S}=I_{8n}} = -8(1+\alpha)^2\,, \tag{2.7}$$

i.e. the three-dimensional AdS radius L_0 is related to the gauge coupling constant g by

$$L_0 = \frac{1}{4|(1+\alpha)g|}\,, \qquad \text{with} \quad R_{\mu\nu} = -4g^2 V_0 G_{\mu\nu} = \frac{2}{L_0^2} G_{\mu\nu}\,. \tag{2.8}$$

We will later talk about AdS spaces with other radii L, but L_0 will always denote the radius at $\mathcal{S} = I_{8n}$, which should be set to unity to obtain correctly normalized CFT correlators (see [7, section 2] for a careful discussion of this point). Note that an $S^3 \times S^3$ compactification corresponds to positive values of α — as can be seen from (2.8), the theory with $\alpha = -1$ admits a Minkowski solution, which we shall not further discuss here. The AdS_3 supersymmetric solution has background isometry group

$$D^1(2,1;\alpha)_L \times D^1(2,1;\alpha)_R\,, \tag{2.9}$$

an $N = (4,4)$ superextension of the three-dimensional AdS group $\mathrm{SU}(1,1)_L \times \mathrm{SU}(1,1)_R$, with the parameter α from (2.4). The spectrum around this local maximum may be organized in representations of (2.9). Following [11], we denote a short supermultiplet of (2.9) by $(\ell_L^+, \ell_L^-; \ell_R^+, \ell_R^-)_S$, where the $\ell_{L,R}^{\pm}$ refer to the SU(2) quantum numbers of the highest weight state in the multiplet under the following bosonic subgroup of (2.9):

$$\left(\mathrm{SU}(2)_L^+ \times \mathrm{SU}(2)_L^-\right) \times \left(\mathrm{SU}(2)_R^+ \times \mathrm{SU}(2)_R^-\right). \tag{2.10}$$

The highest weight state state saturates the bound due to unitarity

$$h \geq \gamma \ell_L^- + (1-\gamma)\ell_L^+ \tag{2.11}$$

where h is the conformal dimension h_L. In this notation, the matter multiplets of (2.1) take the form $(1/2,0;1/2,0)_S$ or $(0,1/2;0,1/2)_S$, each containing 8 scalar and 8 fermionic fields. Their masses may be computed from linearizing the lagrangian (2.1) around the local maximum $\mathcal{S} = I_{8n}$. Metric, gravitini and vector fields are assembled into the "nonpropagating supermultiplets" $(1/2,1/2;0,0)_S$ and $(0,0;1/2,1/2)_S$. The complete list of the appearing supermultiplets, their decomposition into states under (2.10), their masses in (2.1) and the conformal weights $(h,\bar{h})$ under the AdS_3 part of (2.9) are collected in table 1. We used the standard relations (2.26), (2.27) to find $\Delta = h + \bar{h}$ from $m^2 L_0^2$.

This exactly reproduces the spectrum of lowest mass states in the reduction of nine-dimensional supergravity on $AdS_3 \times S^3 \times S^3$, found in [11].[4] More precisely, the infinite sums given in [11] contain among their lowest mass multiplets one $(1/2,0;1/2,0)_S$ and one $(0,1/2;0,1/2)_S$, corresponding to the scalar spectrum of (2.1) with $n = 2$ matter multiplets coupled.

[4]Note that our ordering of SU(2) quantum numbers (3rd column in table 1) differs from that of ref. [11, 1st column] by a permutation.

$(\ell_L^+,\ell_L^-;\ell_R^+,\ell_R^-)_S$	Fields	$(\ell_L^+,\ell_R^+,\ell_L^-,\ell_R^-)$	$(h,\bar h)$	Δ	$m^2L_0^2$
$(\frac{1}{2},\frac{1}{2};0,0)_S$	graviton	$(0,0,0,0)$	$(2,0)$	$\Delta=2$	0
	gravitini	$(\frac{1}{2},0,\frac{1}{2},0)$	$(\frac{3}{2},0)$	$\Delta=\frac{3}{2}$	$\frac{1}{4}$
	vectors	$(1,0,0,0)$	$(1,0)$	$\Delta=1$	0
		$(0,0,1,0)$	$(1,0)$	$\Delta=1$	0
$(0,0;\frac{1}{2},\frac{1}{2})_S$	graviton	$(0,0,0,0)$	$(0,2)$	$\Delta=2$	0
	gravitini	$(0,\frac{1}{2},0,\frac{1}{2})$	$(0,\frac{3}{2})$	$\Delta=\frac{3}{2}$	$\frac{1}{4}$
	vectors	$(0,1,0,0)$	$(0,1)$	$\Delta=1$	0
		$(0,0,0,1)$	$(0,1)$	$\Delta=1$	0
$(0,\frac{1}{2};0,\frac{1}{2})_S$	scalars	$(\frac{1}{2},\frac{1}{2},0,0)$	$(\frac{\alpha}{2(1+\alpha)},\frac{\alpha}{2(1+\alpha)})$	$\Delta_-=1-\frac{1}{1+\alpha}$	$-\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$
		$(0,0,\frac{1}{2},\frac{1}{2})$	$(\frac{1+2\alpha}{2(1+\alpha)},\frac{1+2\alpha}{2(1+\alpha)})$	$\Delta_+=1+\frac{\alpha}{1+\alpha}$	$-\frac{1+2\alpha}{(1+\alpha)^2}$
	fermions	$(\frac{1}{2},0,0,\frac{1}{2})$	$(\frac{\alpha}{2(1+\alpha)},\frac{1+2\alpha}{2(1+\alpha)})$	$\Delta=\frac{1+3\alpha}{2(1+\alpha)}$	$\frac{(1-\alpha)^2}{4(1+\alpha)^2}$
		$(0,\frac{1}{2},\frac{1}{2},0)$	$(\frac{1+2\alpha}{2(1+\alpha)},\frac{\alpha}{2(1+\alpha)})$	$\Delta=\frac{1+3\alpha}{2(1+\alpha)}$	$\frac{(1-\alpha)^2}{4(1+\alpha)^2}$
$(\frac{1}{2},0;\frac{1}{2},0)_S$	scalars	$(\frac{1}{2},\frac{1}{2},0,0)$	$(\frac{2+\alpha}{2(1+\alpha)},\frac{2+\alpha}{2(1+\alpha)})$	$\Delta_+=1+\frac{1}{1+\alpha}$	$-\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$
		$(0,0,\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2(1+\alpha)},\frac{1}{2(1+\alpha)})$	$\Delta_-=1-\frac{\alpha}{1+\alpha}$	$-\frac{1+2\alpha}{(1+\alpha)^2}$
	fermions	$(\frac{1}{2},0,0,\frac{1}{2})$	$(\frac{2+\alpha}{2(1+\alpha)},\frac{1}{2(1+\alpha)})$	$\Delta=\frac{3+\alpha}{2(1+\alpha)}$	$\frac{(1-\alpha)^2}{4(1+\alpha)^2}$
		$(0,\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2(1+\alpha)},\frac{2+\alpha}{2(1+\alpha)})$	$\Delta=\frac{3+\alpha}{2(1+\alpha)}$	$\frac{(1-\alpha)^2}{4(1+\alpha)^2}$

Table 1: Lowest multiplets in the spectrum on $AdS_3\times S^3\times S^3$.

2.2 Truncation and extrema of the potential

We now search for extremal points of the scalar potential (2.6). The dimension of the scalar manifold is $8n$; it is convenient to begin by explicitly fixing all remaining symmetries of the lagrangian (2.1). These symmetries consist of a local $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry and a global $\mathrm{SO}(n)$ symmetry rotating the matter multiplets. For $n=4$ matter multiplets, say, the scalar potential actually only depends on $8\cdot 4-3\cdot 6=14$ parameters out of the original 32. We have computed this scalar potential analytically, and — with some numerical help — found some of its extremal points, see appendix A for a collection of results. These results indicate that a minimal number of $n=3$ matter multiplets is required for the potential to exhibit nontrivial extremal points.

For our main examples, we will further truncate the theory. Following the standard argument [21], extremal points found in the truncation of the scalar sector to singlets under a subgroup of the symmetry group may consistently be lifted to extrema of the full theory. We are mainly interested in extrema which preserve part of the $N=(4,4)$ supersymmetry. The representation content of the scalar sector (table 1) shows that there are no singlets under any nontrivial product of subgroups of the left and right R-symmetry groups $(\mathrm{SU}(2)^+\times\mathrm{SU}(2)^-)_{L,R}$ in (2.10). Thus, at most $N=(1,1)$ supersymmetry can be preserved upon switching on scalar fields.

Assuming $n\geq 4$, we consider the following subgroup

$$G_{\mathrm{inv}}=\mathrm{SU}(2)_{\mathrm{inv}}\times\mathrm{SO}(n-4)\subset\big(\mathrm{SO}(4)^+\times\mathrm{SO}(4)^-\big)\times\big(\mathrm{SO}(4)\times\mathrm{SO}(n-4)\big)$$
$$\subset\big(\mathrm{SO}(4)^+\times\mathrm{SO}(4)^-\big)\times\mathrm{SO}(n)\,, \qquad (2.12)$$

of the global invariance group of the potential (2.6). The $SU(2)_{\rm inv}$ factor in $G_{\rm inv}$ is embedded as the diagonal of the six SU(2) factors on the right hand side. We find that under $G_{\rm inv}$ the scalar spectrum decomposes as

$$4\cdot(\mathbf{1},\mathbf{1})+6\cdot(\mathbf{3},\mathbf{1})+2\cdot(\mathbf{5},\mathbf{1})+2\cdot(\mathbf{1},\mathbf{n-4})+2\cdot(\mathbf{3},\mathbf{n-4})\,. \tag{2.13}$$

Let us study the potential on the four-dimensional space of singlets under $G_{\rm inv}$. This truncation corresponds to restricting the scalar sector to $SO(8,n)$ matrices

$$\begin{gathered}\mathcal{S} = \begin{pmatrix} \cosh B & 0_{4\times(n-4)} & \sinh B & 0_{4\times 4} \\ 0_{(n-4)\times 4} & I_{n-4} & 0_{(n-4)\times 4} & 0_{(n-4)\times 4} \\ \cos A\sinh B & 0_{4\times(n-4)} & \cos A\cosh B & \sin A \\ -\sin A\sinh B & 0_{4\times(n-4)} & -\sin A\cosh B & \cos A \end{pmatrix}, \\ A = \mathrm{diag}\,(p_1,p_2,p_3,p_4)\,, \qquad B = \mathrm{diag}\,(q_1,q_2,q_3,q_4)\,,\end{gathered} \tag{2.14}$$

and further setting $p_2=p_3=p_4$, $q_2=q_3=q_4$. As another (mainly technical) simplification we will from now on set $\alpha=1$. Substituting (2.14) into (2.6), one obtains the potential

$$\begin{aligned} V \;=\; & -16-4\sum_i x_i^2-4\sum_i y_i^2+4\sum_{i<j<k} x_i^2x_j^2x_k^2+4\sum_{i<j<k} y_i^2y_j^2y_k^2+ \\ & +8\prod_i x_i^2+8\prod_i y_i^2+16\prod_i x_i^2y_i^2-16\prod_i\sqrt{1+x_i^2+y_i^2}\,, \end{aligned} \tag{2.15}$$

where the indices i, j, k in the sums and products run from 1 to 4, and we have set

$$x_i=\cos p_i\sinh q_i\,,\qquad y_i=\sin p_i\sinh q_i\,. \tag{2.16}$$

The kinetic term in (2.1) takes the form

$$\begin{aligned} \frac14\sqrt{G}\mathcal{P}_\mu^{Ir}\mathcal{P}^{\mu Ir} &= \frac{\sqrt{G}}{4}\sum_{i=1}^4\left(\sinh^2 q_i(\partial_\mu p_i)^2+(\partial_\mu q_i)^2\right) \\ &= \frac{\sqrt{G}}{4}\sum_{i=1}^4\frac{(1+y_i^2)(\partial_\mu x_i)^2+(1+x_i^2)(\partial_\mu y_i)^2-2x_iy_i(\partial_\mu x_i)(\partial^\mu y_i)}{1+x_i^2+y_i^2}\,. \end{aligned}$$

Linearizing the scalar field equations around the origin gives rise to

$$\Box x_i=\frac{3}{4L_0^2}x_i\,,\qquad \Box y_i=\frac{3}{4L_0^2}y_i\,, \tag{2.17}$$

in agreement with table 1. In particular, all scalar fields satisfy the Breitenlohner-Freedman bound [22], which in these conventions is given by $m^2L^2\geq -1$ for an AdS scale of L. As an illustration of the scalar potential (2.15), we have depicted contour plots of particular slices in figures 1 and 2.

These figures exhibit the most interesting extrema in this truncation of the theory. Details of these extrema are collected in table 2, in particular the remaining gauge — and supersymmetry that they preserve. The unbroken supersymmetries are encoded in the eigenvalues of the tensor A_1 from (2.6), evaluated at the extremum [18, 23]. The number

JHEP05(2002)006

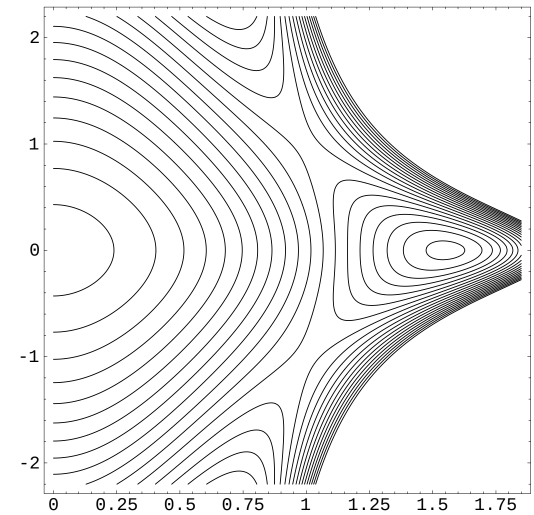

Figure 1: Contour plot $V(x_1, x_2)$, slice: $x_1 = y_1$, $x_2 = y_2$; x_2 is vertical.

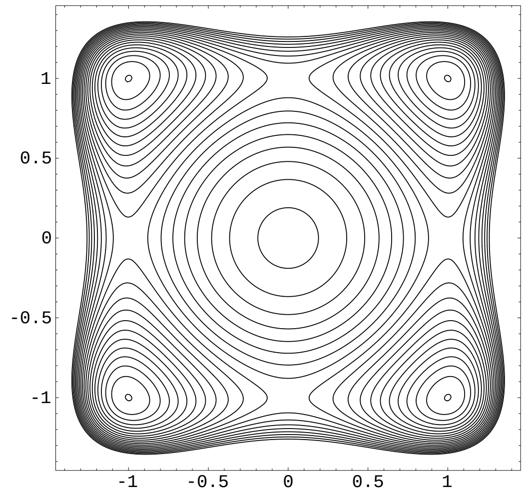

Figure 2: Contour plot $V(x_1, y_1)$, slice: $x_1 = x_2$, $y_1 = y_2$; y_1 is vertical.

(x_1, x_2, y_1, y_2)	# multiplets n	unbroken gauge symmetry	unbroken supersymmetry	central charge $c_{\mathrm{IR}}/c_{\mathrm{UV}}$
$a)(0,0,0,0)$	0	SO(4)×SO(4)	$N=(4,4)$	1
$b)(z_0,0,z_0,0)$	3	$--$	$N=(1,1)$	$\sqrt{2}-1$
$c)(1,1,0,0)$	4	SO(4)	$--$	2/3
$d)(1,1,1,1)$	4	$--$	$N=(1,1)$	1/2

Table 2: Stable extrema with some remaining symmetry, $z_0 = \sqrt{\sqrt{2}+1}$

of preserved supersymmetries coincides with the eigenvalues whose absolute value satisfies $|a_A| = 1/(2gL)$ with the AdS radius L given by $L = 1/\sqrt{2V_{\mathcal{S}=\mathcal{S}_{\mathrm{IR}}}}$. The ratio of the central charge of the associated conformal field theory and the CFT at the origin is given by [24]

$$\frac{c_{\mathrm{IR}}}{c_{\mathrm{UV}}} = \sqrt{\frac{V_0}{V|_{\mathcal{S}=\mathcal{S}_{\mathrm{IR}}}}}\,. \tag{2.18}$$

All of these extrema are stable, i.e. upon linearizing the potential around any of these extremal points, all $8n$ scalars satisfy the Breitenlohner-Freedman bound $m^2L^2 \geq -1$. For the extrema a), b), and d), this is simply a consequence of the unbroken supersymmetry; for the non-supersymmetric extremum c), this can be verified by explicit computation of the scalar fluctuations around the extremum, see appendix A.

Although we have given these results for the particular value $\alpha = 1$ only, the qualitative features of the potential and its extrema remain essentially unchanged for arbitrary α. Upon decreasing the value of α, the shape of figure 1 does not change significantly, while figure 2 is stretched along its vertical axis, breaking the $\mathbb{Z}_4$ symmetry. In the limit $\alpha \to 0$, the minima in the corners of figure 2 (which correspond to the supersymmetric extremum d) in table 2) disappear to infinity. The top and bottom saddle points of figure 2 also move off to infinity, whereas the ones on left and right (which for $\alpha = 1$ give the non-supersymmetric

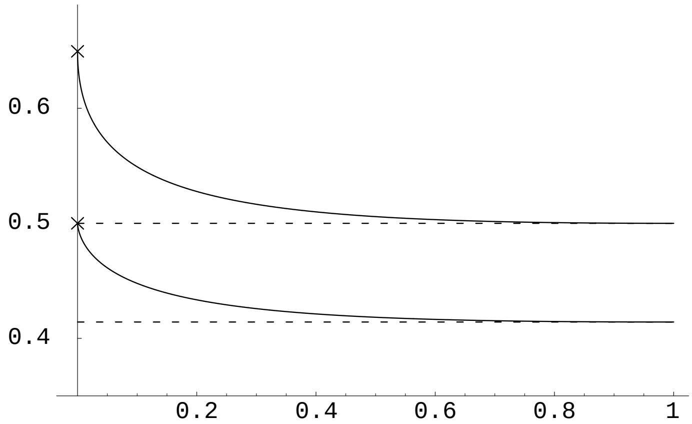

Figure 3: Ratios of central charges as functions of α, see (2.18).

extremum *c*) of table 2) remain critical points of the potential, but become unstable below a certain critical value of α. The other supersymmetric extremum *b*) behaves similarly under change of α.

Summarizing, the two $N = (1,1)$ supersymmetric extrema in table 2 have analogues for any value of $\alpha > 0$, whereas the precise sense of the limit $\alpha \to 0$ requires further investigation. The value of the potential at these extrema changes as a function of α, and so does its ratio to the value of the potential at the origin (2.7). We refrain from including the somewhat lengthy explicit formulas here; as an illustration, the ratios of the central charges of the associated IR boundary theories to the central charge of the UV theory at the origin (2.18) are plotted in figure 3 for the two supersymmetric extrema as a function of α. At $\alpha = 1$, these ratios give the values listed in table 2. In the limit $\alpha \to 0$ they tend to $1/2$ and $3\sqrt{3}/8$, respectively. These values are never taken as both extrema are absent in the theory obtained by naively setting $\alpha = 0$. The physical meaning of the limit $\alpha \to 0$ thus remains to be fully understood. Recall that in the double D1-D5 system this limit corresponds to sending the charges of one of the systems to infinity.

2.3 The supersymmetric extremum

In the following we will mainly study the $N = (1,1)$ supersymmetric extremum *d*) from table 2, which appears as a saddle point in the slice of figure 1 and as a minimum in the upper right corner in figure 2. The value of the potential at this point is $V = -128$, such that the AdS radius is given by $L = 1/(16g)$ (for definiteness we take $g > 0$), and the ratio of central charges is $c_{\rm IR}/c_{\rm UV} = 1/2$. As a first step we will compute the spectrum of physical fields and their masses around this extremum. The matrix $\mathcal{S}$ at this point takes the form

$$\mathcal{S}_{\rm IR} = \begin{pmatrix} \sqrt{3}I_4 & 0_{4\times(n-4)} & \sqrt{2}I_4 & 0_{4\times 4} \\ 0_{(n-4)\times 4} & I_{n-4} & 0_{(n-4)\times 4} & 0_{(n-4)\times 4} \\ I_4 & 0_{4\times(n-4)} & \sqrt{\frac{3}{2}}I_4 & \sqrt{\frac{1}{2}}I_4 \\ -I_4 & 0_{4\times(n-4)} & -\sqrt{\frac{3}{2}}I_4 & \sqrt{\frac{1}{2}}I_4 \end{pmatrix}, \tag{2.19}$$

This preserves a larger subgroup than (2.12), namely

$$\begin{aligned} G_{\rm inv} &= {\rm SO}(4)_{\rm inv} \times {\rm SO}(n-4) \\ &\subset \big({\rm SO}(4)^+ \times {\rm SO}(4)^-\big) \times \big({\rm SO}(4) \times {\rm SO}(n-4)\big)\,, \end{aligned} \tag{2.20}$$

where ${\rm SO}(4)_{\rm inv}$ is embedded as the diagonal of the three ${\rm SO}(4)$ factors on the right hand side. Note that this is a *global* remaining symmetry, i.e. the gauge group (2.2) is completely broken at this extremum, and in agreement with the discussion above there is no R-symmetry of the associated $N = (1,1)$ superconformal field theory. Nevertheless, the spectrum may be organized under (2.20). With respect to ${\rm SO}(4)_{\rm inv}$, or rather $({\rm SU}(2) \times {\rm SU}(2))_{\rm inv}$, the physical fields decompose according to

$$\begin{aligned} \text{gravitons}: &\quad 2\cdot(\mathbf{1},\mathbf{1}) \\ \text{gravitini}: &\quad 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) \\ \text{vectors}: &\quad 2\cdot\big((\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})\big) \\ \text{scalars}: &\quad 2\cdot\big((\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{3}) + (n-4)\cdot(\mathbf{2},\mathbf{2})\big) \\ \text{fermions}: &\quad 2\cdot\big((\mathbf{1},\mathbf{1}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{3}) + (n-4)\cdot(\mathbf{2},\mathbf{2})\big)\,. \end{aligned} \tag{2.21}$$

The scalar masses around this extremum are obtained from computing the fluctuations of the potential (2.6) around the point (2.19). As a result, we find that the two singlets in (2.21) acquire masses

$$m^2L^2 = (5/4,\ 21/4)\,. \tag{2.22}$$

The masses of the scalars in the $(\mathbf{3},\mathbf{3})$ turn out to satisfy $m^2L^2 = -3/4$, i.e. expressed in AdS units they coincide with the masses around the central maximum, cf. table 1 and (2.17) — however, the AdS units have of course changed with respect to $\mathcal{S} = I_{8n}$, the scale has shrunk to half: $L = L_0/2$. Half of the scalars in the $(\mathbf{2},\mathbf{2})$ as well as those in the $(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$ representations become massless around (2.19); the latter are the 12 Goldstone bosons associated with the complete breaking of the gauge symmetry. The other half of the $(\mathbf{2},\mathbf{2})$ scalars turns out to saturate the Breitenlohner-Freedman bound $m^2L^2 = -1$.

The vector fields satisfy first order (i.e. massive self-duality) equations of motion due to their Chern-Simons coupling (2.3), the kinetic scalar term $1/4\sqrt{G}G^{\mu\nu}\mathcal{P}^{Ir}_\mu\mathcal{P}^{Ir}_\nu$ serving as a mass term. Evaluating the latter at (2.19) and diagonalizing the resulting first order equations to bring them to the form [25]

$$\epsilon^{\mu\nu\rho}F_{\mu\nu} = 2m\sqrt{G}B^\rho\,, \tag{2.23}$$

we find that six of the vector fields come with masses $mL = 3/2$, and the other six with masses $mL = 1/2$.

The calculation of the fermion masses finally requires the explicit form of the fermionic part of the lagrangian (2.1) which is, up to quartic terms, given by [20]

$$\begin{aligned} \mathcal{L}_F = {}& \frac{1}{2}\epsilon^{\mu\nu\rho}\overline{\psi}{}^A_\mu D_\nu\psi^A_\rho - \frac{1}{2}i\sqrt{G}\,\overline{\chi}{}^{\dot{A}r}\gamma^\mu D_\mu\chi^{\dot{A}r} - \frac{1}{2}\sqrt{G}\mathcal{P}^{Ir}_\mu\overline{\chi}{}^{\dot{A}r}\Gamma^I_{A\dot{A}}\gamma^\nu\gamma^\mu\psi^A_\nu + \\ & + \frac{1}{2}g\sqrt{G}A_1^{AB}\overline{\psi}{}^A_\mu\gamma^{\mu\nu}\psi^B_\nu + ig\sqrt{G}A_2^{A\dot{A}r}\overline{\chi}{}^{\dot{A}r}\gamma^\mu\psi^A_\mu + \frac{1}{2}g\sqrt{G}A_3^{\dot{A}r\dot{B}s}\overline{\chi}{}^{\dot{A}r}\chi^{\dot{B}s}\,. \end{aligned} \tag{2.24}$$

The gravitini here are denoted by ψ_μ^A, the matter fermions by $\chi^{\dot{A}r}$, indices $A, B, \ldots$ and $\dot{A}, \dot{B}, \ldots$ denote the spinor and conjugate spinor representation, respectively, of the double cover of SO(8). The scalar tensor $A_3^{\dot{A}r\dot{B}s}$ is defined similar to A_1 and A_2 in (2.6) above, see [20] for details. The gravitini masses in (2.24) are extracted from the eigenvalues of the tensor A_1^{AB}. Evaluating A_1 at (2.19) shows that two of the gravitini have masses $mL = 1/2$, corresponding to the two unbroken supersymmetries at this extremum, while the other six become massive Rarita-Schwinger fields with $mL = 1$.

The computation of the fermion masses is slightly more involved. They are essentially encoded in the tensor $A_3^{\dot{A}r\dot{B}s}$; however to properly take care of the super-Higgs effect and the mass-mixing term in the lagrangian, one must first fix the six broken supersymmetry parameters by eliminating six of the fermion fields via

$$\delta\chi^{\dot{A}r} = A_2^{A\dot{A}r}\epsilon^A\,, \tag{2.25}$$

see [12] for a complete discussion. Applied to our case, we obtain fermion masses $mL = \pm 2$ for the two singlets in (2.21), and masses $mL = \pm 1$ for one copy of the $(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$ while the other becomes massless due to the super Higgs effect. The fermions in the $(\mathbf{3},\mathbf{3})$ also become massless, whereas the $(\mathbf{2},\mathbf{2})$ fermions come with masses $mL = \pm 1/2$.

We summarize the results of this section in table 3. The physical fields are organized under the global $\mathrm{SO}(4)_{\mathrm{inv}}$ symmetry (2.20) and grouped into supermultiplets under the $N = (1, 1)$ superconformal symmetry on the boundary. The translation between masses in three dimensions and conformal dimensions on the boundary is given by

$$\Delta_\pm = 1 \pm \sqrt{1 + m^2L^2}\,, \tag{2.26}$$

for the scalar fields [3], and

$$\Delta = 1 + |m|L\,, \tag{2.27}$$

for Rarita-Schwinger fields, matter fermions and vector fields with Chern-Simons action [26, 27, 28, 12]. As in earlier work [12], we choose the sign in (2.26) in accordance with the group-theoretical structure.

The super-Higgs effect occurs in the $(\mathbf{1},\mathbf{3})$ and $(\mathbf{3},\mathbf{1})$ where the 12 scalars and six of the matter fermions become massless (and have not been included in table 3), while the vectors and the gravitino fields acquire mass.

We emphasize that for this extremum not only the central charge but also all conformal dimensions in the dual field theory, computed from (2.26), (2.27), come out to be rational. Of course, for a finite number of primaries, rational charges and weights follow, but one would not *a priori* expect there to only be a finite number of primaries — in fact, as can be clearly seen in the appendix, the conformal weights are typically irrational also here in two dimensions, just as in higher-dimensional examples.

3. The kink solution

We will now construct the kink solution of the gauged supergravity (2.1) that interpolates between the central maximum $\mathcal{S} = I_{8n}$ and the supersymmetric extremum $\mathcal{S} = \mathcal{S}_{\mathrm{IR}}$ (2.19).

$SO(4)_{\rm inv}$	fields	m^2L^2	$(h,\bar{h})$
(1,1)	scalars	$\frac{5}{4},\frac{21}{4}$	$(\frac{7}{4},\frac{7}{4})+(\frac{5}{4},\frac{5}{4})$
	fermions	4	$(\frac{7}{4},\frac{5}{4})+(\frac{5}{4},\frac{7}{4})$
(3,3)	scalars	$-\frac{3}{4}$	$(\frac{1}{4},\frac{1}{4})+(\frac{3}{4},\frac{3}{4})$
	fermions	0	$(\frac{1}{4},\frac{3}{4})+(\frac{3}{4},\frac{1}{4})$
(1,3)	fermions	1	$(\frac{3}{4},\frac{5}{4})$
	vectors	$\frac{1}{4},\frac{9}{4}$	$(\frac{1}{4},\frac{5}{4})+(\frac{3}{4},\frac{7}{4})$
	massive gravitini	1	$(\frac{1}{4},\frac{7}{4})$
(3,1)	fermions	1	$(\frac{5}{4},\frac{3}{4})$
	vectors	$\frac{1}{4},\frac{9}{4}$	$(\frac{5}{4},\frac{1}{4})+(\frac{7}{4},\frac{3}{4})$
	massive gravitini	1	$(\frac{7}{4},\frac{1}{4})$
(2,2)	scalars	$-1,0$	$(1,1)+(\frac{1}{2},\frac{1}{2})$
	fermions	$\frac{1}{4}$	$(1,\frac{1}{2})+(\frac{1}{2},1)$

Table 3: Spectrum around the supersymmetric extremum (2.19) in $N=(1,1)$ supermultiplets

As the latter preserves one quarter of the supersymmetry, one is led to search for a solution which preserves $N=(1,1)$ supersymmetry throughout the flow.

For the metric we employ the standard domain wall ansatz

$$ds^2 = e^{2A(r)}\eta_{ij}dx^i dx^j - dr^2\,, \tag{3.1}$$

where η_{ij} is the two-dimensional Minkowski metric. Pure AdS geometry corresponds to linear behavior of $A(r)=r/L$, with AdS radius L, cf. (2.8). For the scalar fields we use the ansatz (2.14) with $p_1=p_2=p_3=p_4=\pi/4$, $q_1=q_2=q_3=q_4=q(r)$. This corresponds to switching on one of the two singlet scalars from (2.21). A scalar that acquires a radial dependence in the flow is usually referred to as an "active" scalar, whereas the scalars that are zero in the background are "inert". In this truncation, the scalar potential (2.15) reduces to a potential for just the active scalar q:

$$V_q = -\frac{1}{16}\left((3+\cosh 2q)^2(21+12\cosh 2q-\cosh 4q)\right)\,, \tag{3.2}$$

while the kinetic term is just $\sqrt{G}G^{\mu\nu}\partial_\mu q\partial_\nu q$. Straightforward calculation shows that the tensor A_1 from (2.6) can be diagonalized with q-independent eigenvectors, to take the form

$$\begin{aligned} A_1 &= \operatorname{diag}(-X,-X,-X,X,X,X,-W,W)\,,\\ X &= \left(1+\cosh^2 q\right)^2,\\ W &= -\frac{1}{8}(13+20\cosh 2q-\cosh 4q)\,. \end{aligned} \tag{3.3}$$

As usual, W represents the superpotential of the scalar potential V_0, which may be derived from the former as

$$V_0 = \frac{1}{4}\left(\frac{\partial W}{\partial q}\right)^2 - 2W^2\,. \tag{3.4}$$

We denote the corresponding eigenvectors of A_1 by $v_\pm$:

$$A_1 v_\pm = \pm W v_\pm\,. \tag{3.5}$$

The supersymmetric critical points $\mathcal{S} = I_{8n}$ and $\mathcal{S} = \mathcal{S}_{\rm IR}$ (2.19) of the scalar potential V_0 are also critical points of the superpotential W. In particular, this ensures that these points are non-perturbatively stable [29, 30]. The supersymmetric kink solution is now derived by solving the Killing spinor equations for the gravitino and the matter fermions [20]

$$\begin{aligned} \delta_\varepsilon \psi^A_\mu &= D_\mu \varepsilon^A + ig A_1^{AB} \gamma_\mu \varepsilon^B \stackrel{!}{=} 0\,, \\ \delta_\varepsilon \chi^{\dot A r} &= \left(\frac{1}{2} i \Gamma^I_{A\dot A} \gamma^\mu \mathcal{P}^{Ir}_\mu + g A_2^{A\dot A r}\right) \varepsilon^A \stackrel{!}{=} 0\,. \end{aligned} \tag{3.6}$$

With the following ansatz for the Killing spinor ε

$$\varepsilon = F_+(r) v_+ (1 - i\gamma^r)\eta_0 + F_-(r) v_- (1 + i\gamma^r)\eta_0\,, \tag{3.7}$$

where η_0 denotes a constant real 3d spinor, the second equation from (3.6) reduces to

$$\frac{dq}{dr} = \frac{g}{2}\frac{dW}{dq} = -\frac{g}{4}(10\sinh 2q - \sinh 4q)\,. \tag{3.8}$$

This equation may be *analytically* solved as the root of a cubic equation:

$$\frac{(5-y)(y+1)^2}{(y-1)^3} = c_1 e^{24gr}\,, \qquad \text{where} \quad y = \cosh 2q\,, \tag{3.9}$$

and with integration constant c_1 which may be absorbed into a shift of r and will be set to $c_1 = 2$. This flow runs from the central maximum ($y = 1$) at $r = \infty$ to the supersymmetric extremum (2.19) ($y = 5$) at $r = -\infty$. The behavior of the superpotential W along this flow is depicted in figure 4. Following [31, 12, 32], the holographic C function is defined to be proportional to $-1/W$. We emphasize that in our two-dimensional setting, upon computing 2-point correlation functions for the energy-momentum tensor one may eventually compare this holographic definition to Zamolodchikov's definition of the C function [15].

It remains to solve the first equation in the Killing spinor equations (3.6). Substituting (3.7), we obtain

$$\partial_\mu \varepsilon^A - \frac{1}{4} i\omega_\mu{}^{ab}\epsilon_{abc}\gamma^c\varepsilon^A + igW\gamma_\mu\left(F_+(r)v_+^A(1-i\gamma^r) - F_-(r)v_-^A(1+i\gamma^r)\right)\eta_0 \stackrel{!}{=} 0\,,$$

where the spin connection $\omega_\mu{}^{ab}$ introduces the metric into the equation. This yields

$$dA/dr = -2gW\,, \qquad F_\pm = c_\pm e^{-A/2}\,, \tag{3.10}$$

with constants $c_\pm$. Using (3.8), (3.9), this equation may explicitly be integrated as a function of the scalar $q(r)$:

$$e^{6A(r)} = \frac{(5-y)^4}{2(y+1)(y-1)^6}\,, \tag{3.11}$$

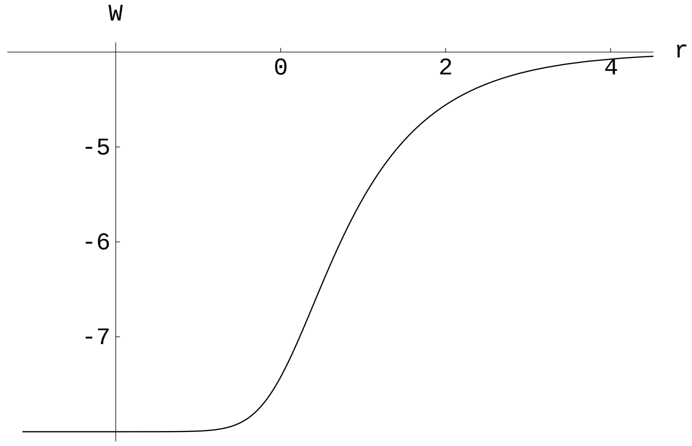

Figure 4: The superpotential W along the kink solution (3.9) (for $g = 1/8$).

where the integration constant has been fixed from asymptotics at $r \to \infty$. Asymptotically, $A(r)$ goes as $8gr$ for $r \to \infty$ and as $16gr$ for $r \to -\infty$, in accordance with the pure AdS behavior of (3.1). That is, setting $g = 1/8$ yields pure AdS with length scale $L_0 = 1$ at $r \to \infty$, which is what one expects from equation (2.8).

To summarize, we have found an analytical solution for the domain wall spacetime given by (3.9), (3.11), admitting two Killing spinors of the form (3.7), which interpolates between the central maximum and the extremum (2.19). Previously constructed holographic flows between conformal endpoints in higher dimensions could only be given numerically [31, 33, 12], preventing the computation of correlation functions in the boundary theories.

To analyze the near-boundary asymptotics of (3.9), it is convenient to introduce the new variable ρ [24]

$$\rho = e^{-2r/L_0} = e^{-16gr}\,, \tag{3.12}$$

such that the line element (3.1) becomes

$$ds^2 = \frac{1}{\rho} e^{2(A(r)-\frac{r}{L_0})} \eta_{ij} dx^i dx^j - \frac{L_0^2}{4\rho^2} d\rho^2\,. \tag{3.13}$$

The asymptotics of the kink solution (3.9), (3.11) close to $\rho = 0$ (i.e. close to the central maximum of the scalar potential) is then given by

$$\begin{aligned} q(\rho) &= \rho^{\frac{1}{4}} \left(1 + \frac{1}{12}\sqrt{\rho} - \frac{13}{160}\rho + \mathcal{O}\left(\rho^{3/2}\right)\right), \\ e^{2A(\rho)} &= \frac{1}{\rho}\left(1 - 2\sqrt{\rho} + \frac{7}{4}\rho + \mathcal{O}\left(\rho^{3/2}\right)\right), \\ W &= -4\left(1 + q^2 + \frac{1}{12}q^4 + \mathcal{O}\left(q^6\right)\right). \end{aligned} \tag{3.14}$$

This behavior of the active scalar field $q(\rho)$ shows that as anticipated in the introduction, this solution — interpreted as a holographic RG flow — corresponds to a deformation of the UV conformal field theory by a relevant operator of dimension $\Delta = 3/2$, rather than to a vev [34]. Indeed, the lowest order power-law behavior of $q(\rho)$ is that expected from standard arguments: $q(\rho) \sim \rho^{(d-\Delta)/2} = \rho^{1/4}$ (cf. equation (2.10) in [7]). We warn the reader that a fair number of expressions in the literature actually degenerate for $d = 2$, the case treated here.

As mentioned in the introduction, the tentative conjecture is that this active scalar is dual to a mass operator for chiral superfields in the large $N = 4$ boundary theory. Let us now make this a little more precise: consider the decoupled worldvolume theory on one D-brane. The spacetime theory on N unordered branes is then the symmetric product of N such theories, i.e. orbifolded by the permutation among the branes. For definiteness, take the two-dimensional large $N = 4$ theory with $c = \tilde{c} = 3$ constructed from one scalar field and four fermions on each chiral side. In $N = 2$ language, and bosonizing one pair of fermions on each side, these fields fill out two hypermultiplets. (Giving vevs to these hypermultiplets would describe motion on the Higgs branch [35, 36], however as pointed out above our flow is not a vev flow). This bosonizing hides half of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ R-symmetry, leaving the diagonal $\mathrm{SU}(2)$ rotating within the hypermultiplets, and the $\mathrm{SU}(2)$ rotating the hypermultiplets into each other. Adding a term which is bilinear in chiral superfields and a singlet under the combined action of these two symmetries, yields a mass term that breaks supersymmetry to $N = (1, 1)$ and the effective theory when the massive fields are integrated out has $c = \tilde{c} = 3/2$. Such a mass term then appears to be the operator that couples to the active scalar field in our flow. The symmetries of this term are consistent with the representation content; our active scalar is a singlet under the remnant symmetry of simultaneously rotating both three-spheres accompanied by a rotation of the matter multiplets. The anomalous dimension $\Delta = 3/2$ cannot be explained by an argument of this type, but one way the conjecture could be checked is if one could find an exact beta function [37] due to the broken conformal symmetry.

A difficult issue in this correspondence is to distinguish, in supergravity, between the bound that short multiplets saturate in representations of near-boundary bulk isometries and the bound in the boundary CFT

$$kh \geq \left(\ell_L^+ - \ell_L^-\right)^2 + k^- \ell_L^+ + k^+ \ell_L^- + u^2 \,, \tag{3.15}$$

where $k = k^+ + k^-$ is the sum of the levels of the two $\mathrm{SU}(2)$ factors, and u is the $\mathrm{U}(1)$ charge. For large k, the nonlinearities disappear from view: setting $\alpha = k^+/k^-$ and considering low-lying ℓ^+, ℓ^- one recovers (2.11). Since the circle in the $S^3 \times S^3 \times S^1$ compactification is taken small in the large N limit, it is difficult to see how one would distinguish states of different $\mathrm{U}(1)$ charge in supergravity.

4. Counterterms and one-point functions

In this section, we compute counterterms for scalars and 1-point functions of their CFT duals following [6, 7, 8]. Together with the fluctuation equations derived in the next section, this in principle yields 2-point functions of the CFT operators throughout the flow. Although correct correlation functions for many operators were obtained long before the aforementioned papers, more difficult cases remained fraught with problems (see e.g. [38]). Progress was reported in [39], followed by the emergence of a coherent picture in [6, 7, 8]. In the current point of view, there are two main ideas to keep in mind; first, the old realization that counterterms are to be introduced on a regulating surface close to the boundary [40, 24, 41, 6]. Second, the addition of finite covariant counterterms, corresponding to a renormalization scheme that preserves supersymmetry [7, 8]. In keeping such a scheme the use of covariant counterterms becomes especially crucial since the latter differ from the non-covariant counterterms in their finite parts. In this paper, we need not worry about finite counterterms since we only discuss uncoupled fluctuations here (inert scalars), but they will become important in correlation functions involving the active scalar [42].

The AdS/CFT correspondence, extended to asymptotically AdS space, posits that CFT correlation functions in the large N, strong coupling limit can be computed from [2, 3]

$$\left\langle e^{\int \hat{\phi}\mathcal{O}} \right\rangle_{\hat{g}} = e^{-S_{\text{sugra}}[\phi, g]} \,, \tag{4.1}$$

where S_{sugra} is the classical supergravity action, and the hatted quantities are the scaled Dirichlet data, i.e. where the (often divergent) dependence on the AdS radius is factored out [5]. (If we would not have taken the supergravity limit, the right hand side would be the full string partition function.) We now proceed to perform the near-boundary analysis (i.e. around $\rho = 0$) for the inert scalars in the kink spacetime. For the order we are interested in, it is most convenient to first linearize their equations of motion.

According to (2.21), under $\mathrm{SO}(4)_{\text{inv}}$ there is one inert scalar in the $(\mathbf{1}, \mathbf{1})$, two copies of the $(\mathbf{3}, \mathbf{3})$, and two times $(n-4)$ copies of the $(\mathbf{2}, \mathbf{2})$. We shall denote these sectors by $\{\mathbf{1}, \mathbf{9}(1), \mathbf{9}(2), \mathbf{4}(1), \mathbf{4}(2)\}$, respectively. We may accordingly express the matrix $\mathcal{S}$ as

$$\mathcal{S} = \mathcal{S}_q \mathcal{S}_\phi \,, \qquad \text{with} \quad \mathcal{S}_\phi = \exp \sum_{\mathbf{i}} \phi^{\mathbf{i}} Y^{\mathbf{i}} \,, \tag{4.2}$$

where $\mathcal{S}_q$ is given by $\mathcal{S}_0$ from (2.14) with $p_i = \pi/4$ and $q_i = q(r)$ while the sum in $\mathcal{S}_\phi$ is a short hand notation for the scalars in the different representations noted above, i.e. the index $\mathbf{i}$ runs over $\{\mathbf{1}, \mathbf{9}(1), \mathbf{9}(2), \mathbf{4}(1), \mathbf{4}(2)\}$ and the $Y^{\mathbf{i}}$ denote the corresponding linear combinations of noncompact generators Y^{Ir}. With this ansatz, the current (2.5) becomes

$$\mathcal{S}^{-1}\partial_\mu \mathcal{S} = \mathcal{S}_\phi^{-1}\partial_\mu \mathcal{S}_\phi + \mathcal{S}_\phi^{-1}\left(\mathcal{S}_q^{-1}\partial_\mu \mathcal{S}_q\right)\mathcal{S}_\phi \,. \tag{4.3}$$

Expanding the kinetic term in lagrangian (2.1) to second order in the inert scalar fluctuations $\phi^{\mathbf{i}}$, thus gives a q-independent kinetic term for the $\phi^{\mathbf{i}}$, whereas the second term in (4.3) contributes to the potential for the $\phi^{\mathbf{i}}$ upon inserting (3.8) for $\mathcal{S}_q^{-1}\partial_\mu \mathcal{S}_q$. Substituting (4.2) and (4.3) into the lagrangian (2.1), leads after some computation to

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\phi \,. \tag{4.4}$$

The different parts of the lagrangian are given by

$$\mathcal{L}_q = \sqrt{G} G^{\mu\nu}\partial_\mu q \partial_\nu q - g^2 \sqrt{G} V_q(q) \,,$$

describing the active scalar q, with the potential V_q from (3.2) above, while the fluctuations of the inert scalars are described by

$$\mathcal{L}_\phi = \frac{1}{4}\sqrt{G}\sum_{\mathbf{i}} G^{\mu\nu}\partial_\mu\phi^{\mathbf{i}}\partial_\nu\phi^{\mathbf{i}} - g^2\sqrt{G}\sum_{\mathbf{i},\mathbf{j}} V_{\mathbf{ij}}(q)\phi^{\mathbf{i}}\phi^{\mathbf{j}}\,. \tag{4.5}$$

Miraculously, the potential $V_{\mathbf{ij}}(q)$ may be diagonalized with q-independent eigenvectors in each of the two-fold degenerate representation sectors **4** and **9**, respectively, such that their equations of motion decouple.[5] Specifically, we find $V_{\mathbf{ij}}(q) = \delta_{\mathbf{ij}}V_{\mathbf{i}}(q)$ with

$$\begin{aligned}
V_{\mathbf{1}} &= \frac{1}{16}\left(-45 - 160y + 10y^2 + 3y^4\right) \\
V_{\mathbf{9}(1)} &= -\frac{1}{4}\left(17 + 30y + y^2\right) \\
V_{\mathbf{9}(2)} &= \frac{1}{16}(y+1)\left(-93 + 13y - 19y^2 + 3y^3\right) \\
V_{\mathbf{4}(1)} &= \frac{1}{16}(y+1)(y-5)\left(17 + 4y + 3y^2\right) \\
V_{\mathbf{4}(2)} &= -\frac{1}{4}(3+y)(7+5y)\,,
\end{aligned} \tag{4.6}$$

where $y = \cosh 2q$, cf. (3.9). The equations of motion for the inert scalar fluctuations implied by (4.4) thus decouple to

$$\partial_\mu\left(\sqrt{G}G^{\mu\nu}\partial_\nu\phi^{\mathbf{i}}\right) = -4g^2\sqrt{G}V_{\mathbf{i}}(q)\phi^{\mathbf{i}}\,. \tag{4.7}$$

We need to expand this equation and the Einstein field equations around $\rho = 0$. As seen in the previous section (3.14), square roots of the radial variable ρ appear in the background; this will also be the case for fluctuations of our scaled Dirichlet data. Square roots are to be expected for a $\Delta = 3/2$ flow in two dimensions, since in general back reaction appears at order $d - \Delta$ for fields dual to relevant operators [6]. To regulate the divergence of the action at $\rho = 0$, we follow the standard prescription of cutting off the bulk integral at $\rho = \epsilon$ and including boundary terms at this radius. Our metric and scalar ansätze read

$$\begin{aligned}
G_{ij}(x,\rho) &= \rho^{-1}g_{ij}(x,\rho)\,, \\
g_{ij}(x,\rho) &= g_{(0)ij}(x) + \sqrt{\rho}\,g_{(1)ij}(x) + \rho g_{(2)ij}(x) + \rho\log\rho h_{(2)ij}(x) + \mathcal{O}\left(\rho^{3/2}\right)\,, \\
\phi(x,\rho) &= \rho^{1/4}\varphi(x,\rho)\,, \\
\varphi(x,\rho) &= \varphi_{(0)}(x) + \sqrt{\rho}\varphi_{(1)}(x) + \sqrt{\rho}\log\rho\psi_{(1)}(x) + \rho\varphi_{(2)}(x) + \mathcal{O}\left(\rho^{3/2}\right)\,.
\end{aligned} \tag{4.8}$$

The logarithmic terms are needed at the given orders, and only there, because the terms at those orders in the naive ansätze are not determined by the equations of motion. The active scalar in the background is

$$q = \rho^{1/4}\mathsf{q} = \rho^{1/4}\left(\mathsf{q}_{(0)} + \sqrt{\rho}\mathsf{q}_{(1)} + \rho\mathsf{q}_{(2)} + \mathcal{O}\left(\rho^{3/2}\right)\right).$$

[5] A similar miracle in [43] was uncovered by appealing to supersymmetry.

We relabel

$$q \to \frac{1}{\sqrt{2}} q\,, \qquad \phi \to \sqrt{2}\phi\,, \qquad V_q, V_{\rm i} \to -V_q, -V_{\rm i}\,, \qquad g_{ij} \to L_0^2 g_{ij}\,, \tag{4.9}$$

and perform a Wick rotation to obtain a canonical lagrangian

$$\sqrt{G}^{-1}\mathcal{L} = \frac{1}{2\kappa^2}R + \frac{1}{2}G^{\mu\nu}\partial_\mu q\partial_\nu q + g^2 V_q + \frac{1}{2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + g^2 V_\phi\,,$$

where we have restored the gravitational coupling κ. Notice that AdS with length scale 1 is recovered by $g^2\kappa^2 = 1/32$. We consider Einstein's equations with the stress-energy tensor

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - G_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + g^2 V_\phi\right),$$

and the same for q. The order $\rho^{-1/2}$ terms yield

$$\mathrm{tr}\,\left(g_{(0)}^{-1}g_{(1)}\right) \;=\; -\kappa^2\left(\mathsf{q}_{(0)}^2 + \varphi_{(0)}^2\right), \tag{4.10}$$

while the $\rho^{-1/2}$ term from (4.7) gives

$$\psi_{(1)} \;=\; -\frac{1}{8}\left(\mathrm{tr}\, g_{(0)}^{-1}g_{(1)}\right)\varphi_{(0)} - \frac{1}{4}\mathsf{q}_{(0)}^2\varphi_{(0)} = \mathcal{O}(\varphi_{(0)}^3)\,. \tag{4.11}$$

Setting $\varphi = 0$, we can check that our background (3.14) indeed satisfies these equations. The scalar curvature R does not appear here; it enters at order ρ^0. We can now write down the regularized on-shell action and apply the fixed-background formalism.

$$\begin{aligned} S_{M,\rm reg} &= -\int_{\rho=\epsilon} d^2x\sqrt{g}\epsilon^{-1/2}\left(\frac{1}{4}\varphi^2 + \epsilon\varphi\partial_\epsilon\varphi\right) \\ &= \int_{\rho=\epsilon} d^2x\sqrt{g_{(0)}}\left(\epsilon^{-1/2}a_{(0)}^{\rm M} - \log\epsilon\, a_{(1)}^{\rm M} + \mathcal{O}(1)\right), \end{aligned} \tag{4.12}$$

where

$$a_{(0)}^{\rm M} = -\frac{1}{4}\varphi_{(0)}^2\,, \qquad a_{(1)}^{\rm M} = \psi_{(1)}\varphi_{(0)}\,.$$

To take all quantities back to the surface at $\rho = \epsilon$, the prescription is to perturbatively invert the relations between $\varphi_{(0)}$ and φ, and $g_{(0)}$ and g. For us, the change from $\sqrt{g_{(0)}}$ to $\sqrt{g}$ only contributes to finite terms. We then have the renormalized action, with the induced metric $\sqrt{\gamma} = \epsilon^{-1}\sqrt{g}$:

$$S_{\rm M,ren} = \lim_{\epsilon\to 0}\left(S_{\rm bulk}(\rho\le\epsilon) + \int_{\rho=\epsilon} d^2x\sqrt{\gamma}\left(\frac{1}{4}\phi^2(x,\epsilon) + \frac{2+\kappa^2}{16}\frac{\log\epsilon}{\epsilon}q^2\phi^2(x,\epsilon)\right)\right).$$

Computing the 1-point function is now simple; there is a contribution both from the regularized action and from the quadratic counterterm

$$\langle\mathcal{O}_\phi\rangle = \lim_{\epsilon\to 0}\frac{1}{\sqrt{g_{(0)}}}\frac{\delta S}{\delta\varphi_{(0)}} = \lim_{\epsilon\to 0}\left(\frac{1}{\epsilon^{3/4}}\frac{1}{\sqrt{\gamma}}\frac{\delta S}{\delta\phi}\right) = -\left(\varphi_{(1)} + \psi_{(1)}\right), \tag{4.13}$$

where we emphasize that we are neglecting finite counterterms. In the background, the inert scalar ϕ is of course zero, so this vev does not constitute a useful check on the computations. A nontrivial check is afforded by the similar computation for the active scalar, including finite counterterms [42].

5. Fluctuation equations

The near-boundary analysis of the previous section is not sufficient to calculate 2-point correlation functions of the associated operators in the boundary theory. In this section we will compute the quadratic fluctuations of the full lagrangian (2.1) around our flow solution (3.9). The 2-point correlation functions may then be extracted from the properly normalized solutions to these equations [38, 43, 7, 44, 8]. As in those cases it turns out that the analysis is most conveniently performed in new so-called horospherical coordinates in which the line element (3.1) takes the form

$$ds^2 = e^{2A(z)}\left(\eta_{ij}dx^i dx^j - dz^2\right), \qquad \text{i.e.}\quad \frac{dz}{dr} = e^{-A}\,. \tag{5.1}$$

For simplicity, we restrict to the fluctuations of inert scalars and vector fields, postponing the active scalar and metric fluctuations to future work. The analysis is greatly simplified by the remaining $SO(4)_{\rm inv}$ symmetry (2.20) which organizes the spectrum. At this linearized level, the scalar fields split into the gauge invariant sectors $(\mathbf{1},\mathbf{1})$, $(\mathbf{3},\mathbf{3})$, and $(\mathbf{2},\mathbf{2})$, whereas the scalars in the $(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$ couple to the vector fields and are shifted under the action of the gauge group (2.2). We treat the two cases separately.

5.1 Inert scalars

We have shown in the previous section, that the equations of motion for the inert scalar fluctuations decouple into second order differential equations (4.7) with potentials (4.6). With the ansatz $\phi^{\mathbf{i}} = e^{-i(p\cdot x)}e^{-A(z)/2}R^{\mathbf{i}}(z)$, the Laplace equations (4.7) turn into

$$\left(-\partial_z^2 + \mathcal{V}_{\mathbf{i}}\right)R^{\mathbf{i}} = p^2 R^{\mathbf{i}}\,, \tag{5.2}$$

with the coordinate z from (5.1) and an effective potential

$$\mathcal{V}_{\mathbf{i}} = \frac{1}{2}A''(z) + \frac{1}{4}A'(z)^2 + 4g^2e^{2A}V_{\mathbf{i}} = e^{2A}g^2\left(3W(q)^2 - \frac{1}{2}W'(q)^2 + 4V_{\mathbf{i}}\right). \tag{5.3}$$

With (3.3), (4.6) we obtain the following effective potentials:

$$\begin{aligned}
\mathcal{V}_{\mathbf{1}} &= \frac{1}{16}g^2e^{2A}(y-1)^2\left(167+34y+7y^2\right)\\
\mathcal{V}_{\mathbf{9}(1)} &= -\frac{5}{16}g^2e^{2A}(y-1)^2(y-5)(y+3)\\
\mathcal{V}_{\mathbf{9}(2)} &= \frac{1}{16}g^2e^{2A}(y-1)^2(y-5)(7y+5)\\
\mathcal{V}_{\mathbf{4}(1)} &= \frac{1}{16}g^2e^{2A}(y-1)^2\left(7+2y+7y^2\right)\\
\mathcal{V}_{\mathbf{4}(2)} &= -\frac{1}{16}g^2e^{2A}(y-1)^2\left(-11-10y+5y^2\right).
\end{aligned} \tag{5.4}$$

Note that according to (3.11), the factor e^{2A} diverges as $(y-1)^{-2}$ near $y=1$, such that all the effective potentials tend to a finite value at the UV boundary. At the other end of the flow ($y=5$) the effective potentials vanish. Their behavior along the flow is depicted in figure 5.

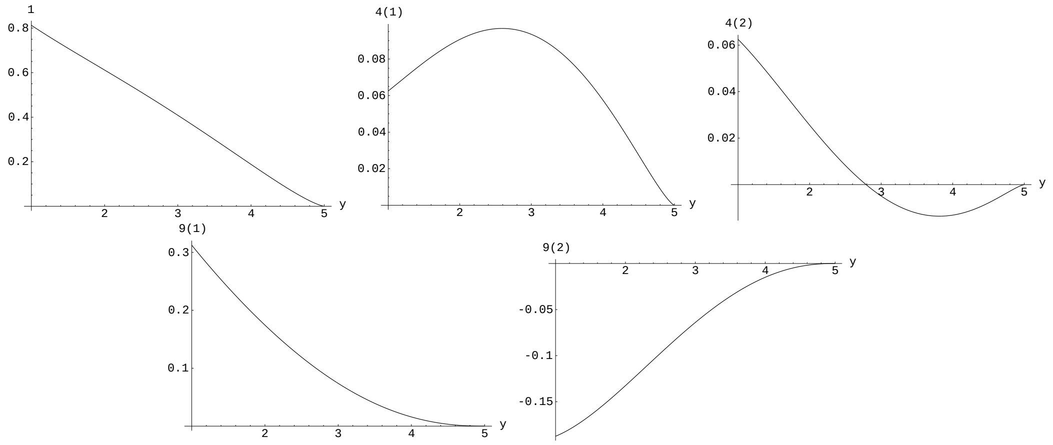

Figure 5: The effective potentials (5.4) of the inert scalar fluctuation equation (5.2), ($g = 1/8$).

Without the $V_{\mathbf{i}}$ contribution, the effective potential (5.3) may obviously be derived from a prepotential in the spirit of supersymmetric quantum mechanics (cf. [38] for a more detailed discussion)

$$\mathcal{V}_{\mathbf{i}} = \mathcal{U}_{\mathbf{i}}' + \mathcal{U}_{\mathbf{i}}^2 \,, \tag{5.5}$$

with $\mathcal{U}_{\mathbf{i}} = 1/2A'(z)$. It has further been noted in [38] that the effective potentials for the active scalar fluctuations in the most prominent exact five-dimensional flows [16, 45, 46] may again be recast into the form (5.5) with modified prepotentials $\mathcal{U}_{\mathbf{i}}$ (though no general prescription has emerged). The absence of tachyonic fluctuations is then manifest.

We find the same result for all effective potentials (5.4); using (3.11), they may be obtained from prepotentials as (5.5) with

$$\begin{aligned}
\mathcal{U}_{\mathbf{1}} &= -\frac{1}{4} g e^A (y-1)(7y-3) \qquad \text{or} \qquad & \mathcal{U}_{\mathbf{1}} &= \frac{1}{4} g e^A (y-1)(y+11) \\
\mathcal{U}_{\mathbf{9}(1)} &= -\frac{1}{4} g e^A (y-1)(y-5) \qquad \text{or} \qquad & \mathcal{U}_{\mathbf{9}(1)} &= -\frac{1}{4} g e^A (y-1)(y+11) \\
\mathcal{U}_{\mathbf{9}(2)} &= \frac{1}{4} g e^A (y-1)(y-5) \\
\mathcal{U}_{\mathbf{4}(1)} &= \frac{1}{4} g e^A (y-1)(y+3) \\
\mathcal{U}_{\mathbf{4}(2)} &= -\frac{1}{4} g e^A (y-1)(y+3) \,.
\end{aligned} \tag{5.6}$$

From (5.5), it follows immediately that a solution to (5.2) which is normalizable as

$$\int_0^\infty |R^{\mathbf{i}}(z)|^2 dz < \infty \,, \tag{5.7}$$

implies $p \geq 0$, i.e. despite their unnerving shapes (cf. figure 5), all the potentials (5.4) have positive spectrum. Note that (5.7) corresponds to the norm $e^{A(z)}dz$ for the original scalar fluctuations $\phi^{\mathbf{i}}$, see [47] for a detailed discussion on the proper choice of the norm.

The fact that the prepotentials $\mathcal{U}_{\mathbf{9}(1)}, \mathcal{U}_{\mathbf{9}(2)}$ and $\mathcal{U}_{\mathbf{4}(1)}, \mathcal{U}_{\mathbf{4}(2)}$ respectively just differ by a sign means that they are superpartners in the sense of supersymmetric quantum mechanics. Specifically, the corresponding solutions of (5.2) may be related by

$$R^{\mathbf{9}(2)} = \left(\partial_z + \mathcal{U}_{\mathbf{9}(2)}\right) R^{\mathbf{9}(1)} \,, \tag{5.8}$$

JHEP05(2002)006

and so on. This may be viewed as a consequence of the fact that $\phi^{\mathbf{9}(1)}$ and $\phi^{\mathbf{9}(2)}$ are part of the same supermultiplet under the remaining $N = (1,1)$ symmetry which governs the flow. It is more surprising that even the prepotential $\mathcal{U}_{\mathbf{1}}$ appears as a superpartner of $\mathcal{U}_{\mathbf{9}(1)}$, i.e. the corresponding potentials have the same spectrum, and their solutions may likewise be mapped onto each other. The inert scalar fluctuation equations with effective potentials (5.4) may thus be reduced to just two independent differential equations.

Consider first $\mathcal{V}_{\mathbf{9}(1)}$. One of its (non-normalizable) zero modes (i.e. solutions of (5.2) with $p = 0$) may be found from

$$\partial_z R_0^{\mathbf{9}(1)} = g e^A (y-5)(y-1)(y+1)\partial_y R_0^{\mathbf{9}(1)} \stackrel{!}{=} \mathcal{U}_{\mathbf{9}(1)} R_0^{\mathbf{9}(1)} , \tag{5.9}$$

which has the solution

$$R_0^{\mathbf{9}(1)} = c_0 (1+y)^{-1/4} . \tag{5.10}$$

Dividing out the zero mode from the general solution $R^{\mathbf{9}(1)} = R_0^{\mathbf{9}(1)}\chi$ leads to

$$\tilde{p}^2 \left(1+t^3\right)\chi(t) - \chi'(t) + t\chi''(t) = 0 , \tag{5.11}$$

with $\tilde{p} = 2^{-4/3}3^{-1/2}p$, $y = \frac{5t^3-1}{1+t^3}$. Exploiting the supersymmetric quantum mechanics structure (5.8), the fluctuation equations for $\mathcal{V}_{\mathbf{1}}$ and $\mathcal{V}_{\mathbf{9}(2)}$ are reduced to the same differential equation. The second differential equation is obtained by similar considerations for $\mathcal{V}_{\mathbf{4}(2)}$. A zero mode to this potential is given by

$$R_0^{\mathbf{4}(2)} = c_0 \frac{(1+y)^{1/12}}{(y-5)^{1/3}} , \tag{5.12}$$

and dividing it out $R^{\mathbf{4}(2)} = R_0^{\mathbf{4}(2)}\chi$ leads to

$$\tilde{p}^2 (1+t^3)\chi(t) + \chi'(t) + t\chi''(t) = 0 , \tag{5.13}$$

with $\tilde{p}$, t defined as above. The two ordinary differential equations (5.11), (5.13) thus contain the entire dynamics of the inert scalar fluctuations.

5.2 Vector/scalar mixing

Let us now consider the sectors in which scalars and vectors are related by the local gauge symmetry. According to (2.21), under $\mathrm{SO}(4)_{\mathrm{inv}}$ these are two copies of the $(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$ which we will denote by $\{\mathbf{6}(1), \mathbf{6}(2)\}$ in the following. The lagrangian in this sector is obtained as in the preceding section from evaluating (2.1) with an ansatz (4.2) where the sum now runs over the corresponding representations $\mathbf{i} \in \{\mathbf{6}(1), \mathbf{6}(2)\}$, including additional contributions from the Chern-Simons and the kinetic scalar term. Again, somewhat miraculously the effective potential may be diagonalized with q-independent eigenvectors, such that the resulting lagrangian is given by

$$\mathcal{L} = \textstyle\sum_{\mathbf{j}} \mathcal{L}_{B^{\mathbf{j}},\phi^{\mathbf{j}}} , \qquad \mathbf{j} \in \{\mathbf{6}(1), \mathbf{6}(2)\} , \tag{5.14}$$

with

$$\mathcal{L}_{B^{\mathbf{j}},\phi^{\mathbf{j}}} = \frac{1}{4}\sqrt{G}G^{\mu\nu}\partial_\mu\phi^{\mathbf{j}}\partial_\nu\phi^{\mathbf{j}} - g^2\sqrt{G}V_{\mathbf{j}}(q)\phi^{\mathbf{j}}\phi^{\mathbf{j}} + \frac{1}{2}\sqrt{G}\epsilon^{\rho\mu\nu}B^{\mathbf{j}}_k F^{\mathbf{j}}_{\mu\nu} + \\ + g\sqrt{G}G^{\mu\nu}B^{\mathbf{j}}_\mu\left(Z_{\mathbf{j}}(q)\partial_\nu\phi^{\mathbf{j}} - \phi^{\mathbf{j}}\partial_\nu Z_{\mathbf{j}}(q)\right) + g^2\sqrt{G}G^{\mu\nu}B^{\mathbf{j}}_\mu B^{\mathbf{j}}_\nu Z_{\mathbf{j}}(q)^2\,, \quad (5.15)$$

and their equations of motion decouple. The effective mass terms $V_{\mathbf{j}}(q)$ and $Z_{\mathbf{j}}(q)$ for scalars and vector fields, respectively, are related by

$$V_{\mathbf{j}} = -\frac{1}{4\sqrt{G}Z_{\mathbf{j}}}\partial_\mu(\sqrt{G}G^{\mu\nu}\partial_\nu Z_{\mathbf{j}})\,, \quad (5.16)$$

with the metric (3.11), (5.1), and explicitly given by

$$V_{\mathbf{6}(1)} = \frac{1}{16}(y+1)(y-5)\left(17+4y+3y^2\right) \\ V_{\mathbf{6}(2)} = \frac{1}{4}(y-5)\left(5+5y+2y^3\right)\,, \quad (5.17)$$

and

$$Z_{\mathbf{6}(1)} = \sqrt{2(y-1)}\,, \qquad Z_{\mathbf{6}(2)} = \sqrt{y^2-1}\,. \quad (5.18)$$

The linearized local gauge symmetry (2.2) acts as

$$\delta B^{\mathbf{j}}_\mu \to \partial_\mu\Lambda^{\mathbf{j}}\,, \qquad \delta\phi^{\mathbf{j}} \to -2gZ_{\mathbf{j}}(q)\Lambda^{\mathbf{j}}\,, \quad (5.19)$$

which leaves (5.15) invariant, as may be explicitly checked making use of (5.16). The equations of motion obtained from variation of (5.15) give the duality equations relating vector and scalar fields

$$\epsilon^{\mu\nu\rho}F^{\mathbf{j}}_{\mu\nu} = -2g\sqrt{G}Z_{\mathbf{j}}(q)^2G^{\rho\nu}\left(B^{\mathbf{j}}_\nu + \frac{1}{2g}\partial_\nu(Z_{\mathbf{j}}(q)^{-1}\phi^{\mathbf{j}})\right)\,, \quad (5.20)$$

and the scalar equation of motion which may be obtained from the Bianchi identities implied by (5.20). We may fix the gauge (5.19) by setting $\phi^{\mathbf{j}}$ to zero, but will prefer to equivalently work with the gauge invariant object

$$\mathcal{B}^{\mathbf{j}}_\mu = B^{\mathbf{j}}_\mu + \frac{1}{2g}\partial_\mu\left(Z_{\mathbf{j}}^{-1}\phi^{\mathbf{j}}\right)\,. \quad (5.21)$$

The duality equation (5.20) then simply takes the form

$$\epsilon^{\rho\mu\nu}F^{\mathbf{j}}_{\mu\nu} = -2g\sqrt{G}Z_{\mathbf{j}}^2G^{\rho\nu}\mathcal{B}^{\mathbf{j}}_\nu\,, \quad (5.22)$$

while the scalar equation of motion gives the current conservation

$$\partial_\mu\left(\sqrt{G}Z_{\mathbf{j}}^2G^{\mu\nu}\mathcal{B}^{\mathbf{j}}_\nu\right) = 0\,. \quad (5.23)$$

Combining these two equations and assuming a metric of conformal form (5.1), leads after some computation to

$$\left(G^{\mu\nu}\partial_\mu\partial_\nu\right)\mathcal{B}^{\mathbf{j}}_\rho = -g^2Z_{\mathbf{j}}^4\mathcal{B}^{\mathbf{j}}_\rho - (\partial_\rho + 2\partial_\rho A)\left(\mathcal{B}^{\mathbf{j}}_\mu G^{\mu\nu}\partial_\nu\left(A + 2\ln Z_{\mathbf{j}}\right)\right) - \\ - F^{\mathbf{j}}_{\rho\mu}G^{\mu\nu}\partial_\nu\left(A + 2\ln Z_{\mathbf{j}}\right)\,. \quad (5.24)$$

With the ansatz $\mathcal{B}^{\mathbf{j}}_z = e^{-i(p\cdot x)} e^{-A(z)/2} Z_{\mathbf{j}}^{-1} b^{\mathbf{j}}_z(z)$, the corresponding component of the vector equations of motion (5.24) turns into

$$\left(-\partial_z^2 + \mathcal{V}_{\mathbf{j}}\right) b^{\mathbf{j}}_z = p^2 b^{\mathbf{j}}_z\,, \tag{5.25}$$

where the effective potentials are found after some computation to be

$$\mathcal{V}_{\mathbf{j}} = g^2 e^{2A} \left(Z_{\mathbf{j}}^4 + \frac{1}{2}\left(\ln' Z_{\mathbf{j}}\right)^2 W'^2 - \frac{W'}{4Z_{\mathbf{j}}}\left(W' Z_{\mathbf{j}}'\right)' + \frac{1}{2} W'^2 - W^2 \right), \tag{5.26}$$

with primes denoting derivatives with respect to q. Inserting the superpotential (3.3) and the scalar functions from (5.18) then yields:

$$\begin{aligned} \mathcal{V}_{\mathbf{6}(1)} &= -\frac{5}{16} g^2 e^{2A} (y-1)^2 (y-5)(y+3) \\ \mathcal{V}_{\mathbf{6}(2)} &= \frac{1}{16} g^2 e^{2A} (y-1)^2 \left(167 + 34y + 7y^2\right). \end{aligned} \tag{5.27}$$

Surprisingly, these effective potentials precisely coincide with the effective potentials for the scalars $\mathcal{V}_{\mathbf{9}(1)}$ and $\mathcal{V}_{\mathbf{1}}$, respectively! In particular, (5.6) shows that $\mathcal{V}_{\mathbf{6}(1)}$ and $\mathcal{V}_{\mathbf{6}(2)}$ admit prepotentials and are superpartners in the sense of supersymmetric quantum mechanics. Following the analysis of the last section, the equations of motion for the longitudinal vector components $\mathcal{B}_z$ thus may again be reduced to the ordinary differential equation (5.11). The transverse components $\mathcal{B}_0$, $\mathcal{B}_1$ are finally obtained from the duality equation (5.20) as explicit functions of $\mathcal{B}_z$ and its derivatives. This is in agreement with the fact that the three-dimensional vectors carry the same number of degrees of freedom as the scalar fields.

This finishes our computation of fluctuation equations in the background (3.9). As discussed above, these equations in principle allow us to compute 2-point functions throughout the flow. We leave this endeavor to future work.

6. Conclusion

We have found an analytic domain wall solution in three-dimensional gauged supergravity which describes an $N = (1,1)$ supersymmetric RG flow between conformal fixed points of a two-dimensional quantum field theory. It is driven by a relevant operator of conformal dimension $\Delta = 3/2$. We have computed counterterms to the lagrangian, and the 1-point functions for inert scalars in the presence of sources. Finally, we have derived the fluctuation equations for inert scalars and vector fields which reduce to the second order differential equations (5.11), and (5.13).

While these differential equations appear fairly simple, they are not of hypergeometric type as were those encountered in higher-dimensional examples previously treated in the literature. So far we have not found analytic solutions to these equations. They would immediately yield the 2-point correlation functions, cf. [38, 43, 7, 44, 8]. However, this information may in principle also be extracted numerically from (5.11), (5.13). We stress that the crucial step in the whole analysis was finding an analytic kink solution, whereas a purely numerical description of this flow would not be sufficient. In contrast, an analytic solution to the fluctuation equations would certainly be helpful and of interest, but is not required for the final computation of correlation functions.[6]

In the course of our discussion, several "miracles" passed before our eyes. The ratio of central charges in the $\alpha = 1$ case comes out rational, just like in the FGPW flow of 3+1 boundary theory [12] — despite being given by a square root formula. For the extremum we study, even the conformal dimensions in the infrared come out rational, something which did not happen in the FGPW flow. Note that the same phenomena occur for the nonsupersymmetric but stable extremum c) from table 2; it has rational central charge as well as rational conformal dimensions, cf. table 5, and might deserve further study.

Further, the lagrangian for the inert scalars and the vector fields was found to be diagonalizable, such that the equations of motion of these fields led to uncoupled second order differential equations. All of these equations admitted simple prepotentials in the sense of supersymmetric quantum mechanics. Even more surprising, we found that despite being scattered on several distinct supermultiplets, all fluctuation equations of motion could be reduced to just two quite innocuous equations (5.11), (5.13). There seems to be no direct reason why the effective potential for the singlet scalar $\mathcal{V}_{\mathbf{1}}$, as well as the effective potential for the longitudinal vector fields $\mathcal{V}_{\mathbf{6}(1)}$, would turn out to be nothing but the superpartners of $\mathcal{V}_{\mathbf{9}(1)}$. From a pure supergravity perspective, the occurrences of rational central charges as well as rational conformal dimensions around the supersymmetric extremum may seem like black magic; they presumably admit rational explanation from the point of view of the holographic renormalization group — lending further support to the latter as an extension of the AdS/CFT correspondence.

As this paper draws to a close, let us give some general ideas of future directions of study. As an immediate application, the 2-point functions of operators dual to inert scalars and vectors should be extracted from the fluctuation equations given here. Two-point functions of the operator $\mathcal{O}_q$ dual to the active scalar q and the stress-energy tensor require some additional calculations but are in principle straightforward to obtain using the formalism of holographic renormalization. This should then settle, among other things, whether the flow presented here is truly a mass deformation. As we emphasized in the introduction, the relatively advanced understanding of 1+1 conformal field theories can then be exploited so that quantities like the C function can actually be directly computed in the dual field theory, at least in principle; of course this task is not entirely trivial when the theory is strongly coupled.

Further, most of our discussion qualitatively applies to general values of α (the ratio of coupling constants in the gauged supergravity, or the ratio of SU(2) levels in the boundary CFT), and other values than our main example $\alpha = 1$ are interesting to study. We picked $\alpha = 1$ as an example partly due to technical simplifications. By turning on scalars at other values of α, we can study RG flows from large $N = 4$ superconformal field theories, driven by operators of dimension $\Delta_+ = 1 + \frac{1}{1+\alpha}$, which for small α approaches marginality. In fact, this limit may be of particular interest; in the double D1-D5 system (see the introduction),

[6]General properties of holographic CFT/CFT flows have recently been studied in [48]. See also [49] for earlier work on model-independent statements on holographic RG flows.

$\alpha \to 0$ corresponds to one of the two D1-D5 system decoupling, a limit with subtleties of its own. We hope that eventually, RG flows in three-dimensional gauged supergravity theories will shed some light on certain aspects of the quantum mechanics of nonextremal black holes.

Another very interesting topic is a chiral breaking of supersymmetry, e.g. $N = (4,4)$ breaking to $N = (4,0)$ in these models. As discussed in the main text, this cannot be accomplished by flows of the type studied here, which turn on nothing but scalar fields. In this context it would be most interesting to understand, within the framework presented here, the higher-dimensional $N = (4,0)$ solution recently constructed in [50].

Finally, let us turn the spotlight to a related scene of interest: the maximal three-dimensional gauged supergravity, with 32 supercharges, that was constructed in [19, 23]. That theory enjoys even closer analogy to the maximal theories used in five dimensions, and while being more involved technically (for instance, scalars parametrizing the exceptional coset manifold $E_{8(8)}/\,\mathrm{SO}(16)$ rather than the orthogonal ones encountered here) it exhibits several very intriguing features.

Acknowledgments

We gratefully acknowledge useful discussions with C. Bachas, P. Bain, J. de Boer, R. McNees, F. Morales, I. Runkel, K. Skenderis, B. Sundborg, M. Trigiante, P. Vanhove, F. Zamora, and we especially thank M. Bianchi and M. Haack for useful comments on early drafts of this paper. M.B. is supported by a Marie Curie Fellowship, contract number HPMF-CT-2001-01311. This work is partly supported by EU contract HPRN-CT-2000-00122.

A. Scalar potential and stable extrema

In this appendix, we give an explicit parametrization of the scalar potential (2.6) for $n = 4$ matter multiplets and collect some of its stable extrema together with the spectrum around these points.

As discussed in the main text, a naive counting suggests that this potential is a function of 14 variables. To begin with, the matrix $\mathcal{S}$ is an element of SO(8, 4). The freedom of right multiplication by compact SO(8) × SO(4) matrices, corresponding to the coset structure of the scalar manifold, may be fixed by bringing $\mathcal{S}$ into the form

$$\mathcal{S} = \begin{pmatrix} \sqrt{I_4 + Y^T Y} & Y^T \\ Y & \sqrt{I_8 + Y Y^T} \end{pmatrix}, \tag{A.1}$$

with an 8×4 matrix Y. In this representation system of the coset manifold, the local $\mathrm{SO}(4) \times \mathrm{SO}(4)$ invariance acts by conjugation on $\mathcal{S}$, i.e. as

$$Y \;\to\; \Lambda_1 Y\,, \qquad \Lambda_1 \in \mathrm{SO}(4) \times \mathrm{SO}(4)\,. \tag{A.2}$$

The global SO(4) symmetry rotating the matter multiplets likewise acts by conjugation on $\mathcal{S}$ such that

$$Y \;\to\; Y \Lambda_2\,, \qquad \Lambda_2 \in \mathrm{SO}(4)\,. \tag{A.3}$$

Fixing (A.3) in the singular value decomposition of Y, allows us to bring it into the form

$$Y = SD_2\,, \qquad S \in \mathrm{SO}(8)\,, \qquad D_2 = \begin{pmatrix} \mathrm{diag}\,(q_1, q_2, q_3, q_4) \\ 0 \end{pmatrix}. \tag{A.4}$$

The SO(8) matrix S may be decomposed into $S = T_1 D T_2$ with block matrices $T_1, T_2 \in \mathrm{SO}(4) \times \mathrm{SO}(4)$, and $D = ((\cos D_1, \sin D_1), (-\sin D_1, \cos D_1))$, with a 4×4 diagonal matrix D_1. Fixing (A.2) to absorb T_1, and changing the representation system (A.1) by a final SO(4) rotation from the right hand side to absorb one of the SO(4) blocks from T_2, we may eventually bring $\mathcal{S}$ into the form

$$\mathcal{S} = \begin{pmatrix} I_4 & & \\ & \cos D_1 & \sin D_1 \\ & -\sin D_1 & \cos D_1 \end{pmatrix} \begin{pmatrix} I_4 & & \\ & T & \\ & & I_4 \end{pmatrix} \begin{pmatrix} \cosh D_2 & \sinh D_2 & \\ \sinh D_2 & \cosh D_2 & \\ & & I_4 \end{pmatrix}, \tag{A.5}$$
$$D_1 = \mathrm{diag}\,(p_1, p_2, p_3, p_4)\,, \qquad D_2 = \mathrm{diag}\,(q_1, q_2, q_3, q_4)\,, \qquad T \in \mathrm{SO}(4)\,.$$

This contains precisely 14 parameters; all the redundancies are fixed. Inserting (A.5) back into (2.6) yields the scalar potential V as explicit function of these 14 variables. We have numerically found some extrema on this space, all of which exhibit $T = I_4$, i.e. live in the truncation (2.14). The extremal points preserving some remaining symmetry have been listed in table 2; *b*) and *d*) preserve $N = (1, 1)$ supersymmetry, while *c*) has an unbroken SO(4) diagonal of the gauge group (2.2). The spectrum of physical fields around *d*) has been given in table 3 and further exploited in the main text. For completeness, we give here the spectra around *b*) and *c*), computed in analogy to table 3. The former one which decomposes into $N = (1, 1)$ supermultiplets is listed in table 4. The latter is organized by the unbroken SO(4) and presented in table 5, it corresponds to a nonsupersymmetric two-dimensional CFT with rational central charge $c_{\mathrm{IR}}/c_{\mathrm{UV}} = 2/3$.

In addition, we have found some extrema which do not preserve any remaining symmetry, but which are stable extremal points of the full potential (2.6), i.e. all $8n$ scalars satisfy the Breitenlohner-Freedman bound [22]. The corresponding boundary theories are non-supersymmetric CFTs with irrational central charge. In coordinates (2.16), these extremal points are located at

- $$x_1 = x_2 = -y_1 = -y_2 = 1\,, \qquad x_3 = -x_4 = y_3 = y_4 = \sqrt{3}\,, \tag{A.6}$$
  with $c_{\mathrm{IR}}/c_{\mathrm{UV}} = 1/\sqrt{7} \;\approx\; 0.3790.$

- $$x_1 = x_2 = x_3 = x_4 = -y_1 = y_2 = y_3 = y_4 = \frac{\sqrt{2+\sqrt{13}}}{\sqrt{3}}\,, \tag{A.7}$$
  with $c_{\mathrm{IR}}/c_{\mathrm{UV}} \;=\; \frac{3\sqrt{3}}{\sqrt{97+26\sqrt{13}}} \;\approx\; 0.3762.$

- $$x_1 = -y_1 = \frac{\sqrt{1+2\sqrt{5}}}{\sqrt{2}}\,, \qquad x_2 = x_3 = x_4 = y_2 = y_3 = y_4 = \frac{\sqrt{1+\sqrt{5}}}{\sqrt{2}}\,, \tag{A.8}$$
  with $c_{\mathrm{IR}}/c_{\mathrm{UV}} \;=\; \frac{4}{\sqrt{57+25\sqrt{5}}} \;\approx\; 0.3765.$

$SU(2)_{\rm inv}$	fields	m^2L^2	$(h,\bar{h})$
1	scalars	$-\frac{1}{2},\frac{11+8\sqrt{2}}{6+4\sqrt{2}}$	$(\frac{1}{2}+\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})+(1+\frac{1}{2\sqrt{2}},1+\frac{1}{2\sqrt{2}})$
	fermions	$\frac{3}{4}+\frac{1}{\sqrt{2}}$	$(\frac{1}{2}+\frac{1}{2\sqrt{2}},1+\frac{1}{2\sqrt{2}})+(1+\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})$
1	scalars	$1,\frac{2(7+5\sqrt{2})}{3+2\sqrt{2}}$	$(\frac{1}{2}+\frac{1}{2}\sqrt{2},\frac{1}{2}+\frac{1}{2}\sqrt{2})+(1+\frac{1}{2}\sqrt{2},1+\frac{1}{2}\sqrt{2})$
	fermions	$\frac{9}{4}+\sqrt{2}$	$(\frac{1}{2}+\frac{1}{2}\sqrt{2},1+\frac{1}{2}\sqrt{2})+(1+\frac{1}{2}\sqrt{2},\frac{1}{2}+\frac{1}{2}\sqrt{2})$
3	scalars	$-1,0$	$(1,1)+(\frac{1}{2},\frac{1}{2})$
	fermions	$\frac{1}{4}$	$(1,\frac{1}{2})+(\frac{1}{2},1)$
3	fermions	$\frac{3}{4}+\frac{1}{\sqrt{2}}$	$(\frac{1}{2}+\frac{1}{2\sqrt{2}},1+\frac{1}{2\sqrt{2}})$
	vectors	$\frac{1}{2},\frac{3}{2}+\sqrt{2}$	$(\frac{1}{2\sqrt{2}},1+\frac{1}{2\sqrt{2}})+(\frac{1}{2}+\frac{1}{2\sqrt{2}},\frac{3}{2}+\frac{1}{2\sqrt{2}})$
	gravitini	$\frac{3}{4}+\frac{1}{\sqrt{2}}$	$(\frac{1}{2\sqrt{2}},\frac{3}{2}+\frac{1}{2\sqrt{2}})$
3	fermions	$\frac{3}{4}+\frac{1}{\sqrt{2}}$	$(1+\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})$
	vectors	$\frac{1}{2},\frac{3}{2}+\sqrt{2}$	$(1+\frac{1}{2\sqrt{2}},\frac{1}{2\sqrt{2}})+(\frac{3}{2}+\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})$
	gravitini	$\frac{3}{4}+\frac{1}{\sqrt{2}}$	$(\frac{3}{2}+\frac{1}{2\sqrt{2}},\frac{1}{2\sqrt{2}})$
5	scalars	$-\frac{1}{2},-\frac{5+4\sqrt{2}}{6+4\sqrt{2}}$	$(\frac{1}{2\sqrt{2}},\frac{1}{2\sqrt{2}})+(\frac{1}{2}+\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})$
	fermions	$\frac{3}{4}-\frac{1}{\sqrt{2}}$	$(\frac{1}{2\sqrt{2}},\frac{1}{2}+\frac{1}{2\sqrt{2}})+(\frac{1}{2}+\frac{1}{2\sqrt{2}},\frac{1}{2\sqrt{2}})$

Table 4: Spectrum around extremum b) (Table 2) in $N=(1,1)$ supermultiplets

It may be noted as an amusing coincidence that their central charges lie within a range of less than one percent. The physical spectra around these extremal points are collected in tables 6–8. We have not included the fields which become massless due to the (super) Higgs effect.

fields	Δ	#
scalars	$\frac{10}{3}$	1
	$1\pm\frac{2}{3}$	16
	$1\pm\frac{1}{3}$	9
fermions	$\frac{7}{6}$	24
vectors	$\frac{7}{3}$	6
gravitini	$\frac{11}{6}$	8

Table 5: Spectrum around c) (table 2).

fields	Δ	#
scalars	$1\pm\frac{\sqrt{2(3\sqrt{13}-5)}}{\sqrt{23}}$	9
	$1+\frac{\sqrt{2(17-\sqrt{13})}}{\sqrt{23}}$	9
	$1+\frac{\sqrt{2(31+9\sqrt{13})}}{\sqrt{23}}$	1
	$1+\frac{\sqrt{2(53+5\sqrt{13})}}{\sqrt{23}}$	1
fermions	1	16
	$1+\frac{\sqrt{71+31\sqrt{13}}}{\sqrt{46}}$	8
vectors	$1+\frac{\sqrt{223+62\sqrt{13}}}{\sqrt{97+26\sqrt{13}}}$	12
gravitini	$1+\frac{\sqrt{31+9\sqrt{13}}}{\sqrt{46}}$	8

Table 6: Spectrum around (A.7)

fields	Δ	#
scalars	$1 \pm \frac{2}{\sqrt{7}}$	6
	$1 \pm \frac{\sqrt{3}}{\sqrt{7}}$	4
	1 ± 1	4
	$1 + \frac{2\sqrt{3}}{\sqrt{7}}$	2
	$1 + \frac{3\sqrt{2}+2}{\sqrt{7}}$	1
	$1 \pm \frac{3\sqrt{2}-2}{\sqrt{7}}$	1
	$1 \pm \frac{\sqrt{22-4\sqrt{21}}}{\sqrt{7}}$	1
	$1 + \frac{\sqrt{22+4\sqrt{21}}}{\sqrt{7}}$	1
fermions	1	12
	3	4
	$1 + \frac{\sqrt{23-3\sqrt{57}}}{\sqrt{14}}$	4
	$1 + \frac{\sqrt{23+3\sqrt{57}}}{\sqrt{14}}$	4
vectors	$1 + \frac{4}{\sqrt{7}}$	8
	$1 + \frac{3}{\sqrt{7}}$	2
	$1 + \sqrt{3}$	2
gravitini	$1 + \frac{2\sqrt{3}}{\sqrt{7}}$	4
	2	4

Table 7: Spectrum around (A.6).

fields	Δ	#
scalars	$1 \pm \frac{\sqrt{-141+70\sqrt{5}}}{\sqrt{31}}$	8
	$1 + \frac{\sqrt{-139+80\sqrt{5}}}{\sqrt{31}}$	5
	1 ± 1	3
	$1 + \frac{\sqrt{-20+55\sqrt{5}+\sqrt{24286-7530\sqrt{5}}}}{\sqrt{31}}$	1
	$1 \pm \frac{\sqrt{-20+55\sqrt{5}-\sqrt{24286-7530\sqrt{5}}}}{\sqrt{31}}$	1
	$1 + \frac{\sqrt{65+15\sqrt{5}+\sqrt{22601-7740\sqrt{5}}}}{\sqrt{31}}$	1
	$1 \pm \frac{\sqrt{65+15\sqrt{5}-\sqrt{22601-7740\sqrt{5}}}}{\sqrt{31}}$	1
fermions	$1 + \frac{\sqrt{-313+140\sqrt{5}}}{\sqrt{62}}$	16
	$1 + \frac{\sqrt{5(-13+28\sqrt{5})}}{\sqrt{62}}$	6
	$1 + \frac{\sqrt{47+80\sqrt{5}}}{\sqrt{62}}$	2
vectors	$1 + \frac{\sqrt{6(23+11\sqrt{5})}}{\sqrt{57+25\sqrt{5}}}$	6
	$1 + \frac{\sqrt{2(59+27\sqrt{5})}}{\sqrt{57+25\sqrt{5}}}$	6
gravitini	$1 + \frac{\sqrt{5(7+4\sqrt{5})}}{\sqrt{62}}$	6
	$1 + \frac{\sqrt{-77+80\sqrt{5}}}{\sqrt{62}}$	2

Table 8: Spectrum around (A.8)

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] V. Balasubramanian and P. Kraus, *Spacetime and the holographic renormalization group*, *Phys. Rev. Lett.* **83** (1999) 3605 [hep-th/9903190].

[5] J. de Boer, E. Verlinde and H. Verlinde, *On the holographic renormalization group*, *J. High Energy Phys.* **08** (2000) 003 [hep-th/9912012].

[6] S. de Haro, S.N. Solodukhin and K. Skenderis, *Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence*, *Commun. Math. Phys.* **217** (2001) 595 [hep-th/0002230].

[7] M. Bianchi, D.Z. Freedman and K. Skenderis, *How to go with an RG Flow*, *J. High Energy Phys.* **08** (2001) 041 [hep-th/0105276].

[8] M. Bianchi, D.Z. Freedman and K. Skenderis, *Holographic renormalization*, hep-th/0112119.

[9] H.J. Boonstra, B. Peeters and K. Skenderis, *Brane intersections, anti-de Sitter spacetimes and dual superconformal theories*, *Nucl. Phys.* **B 533** (1998) 127 [hep-th/9803231].

[10] S. Elitzur, O. Feinerman, A. Giveon and D. Tsabar, *String theory on* $AdS(3) \times S^3 \times S^3 \times S^1$, *Phys. Lett.* **B 449** (1999) 180 [hep-th/9811245].

[11] J. de Boer, A. Pasquinucci and K. Skenderis, *AdS/CFT dualities involving large 2d* $N = 4$ *superconformal symmetry*, *Adv. Theor. Math. Phys.* **3** (1999) 577 [hep-th/9904073].

[12] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, *Adv. Theor. Math. Phys.* **3** (1999) 363 [hep-th/9904017].

[13] J.M. Maldacena, G.W. Moore and A. Strominger, *Counting BPS black holes in toroidal type-II string theory*, hep-th/9903163.

[14] A. Sevrin, W. Troost and A. Van Proeyen, *Superconformal algebras in two-dimensions with* $N = 4$, *Phys. Lett.* **B 208** (1988) 447.

[15] A.B Zamolodchikov, *'Irreversibility' of the flux of the renormalization group in a 2-d field theory*, *JETP Lett.* **43** (1986) 730.

[16] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of* $N = 1$ *super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [hep-th/9909047].

[17] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[18] M. Günaydin, L.J. Romans and N.P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[19] H. Nicolai and H. Samtleben, *Maximal gauged supergravity in three dimensions*, *Phys. Rev. Lett.* **86** (2001) 1686 [hep-th/0010076].

[20] H. Nicolai and H. Samtleben, $N = 8$ *matter coupled* AdS_3 *supergravities*, *Phys. Lett.* **B 514** (2001) 165 [hep-th/0106153].

[21] N.P. Warner, *Some new extrema of the scalar potential of gauged* $N = 8$ *supergravity*, *Phys. Lett.* **B 128** (1983) 169.

[22] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249.

[23] H. Nicolai and H. Samtleben, *Compact and noncompact gauged maximal supergravities in three dimensions*, *J. High Energy Phys.* **04** (2001) 022 [hep-th/0103032].

[24] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *J. High Energy Phys.* **07** (1998) 023 [hep-th/9806087].

[25] P.K. Townsend, K. Pilch and P. van Nieuwenhuizen, *Selfduality in odd dimensions*, *Phys. Lett.* **B 136** (1984) 38.

[26] M. Henningson and K. Sfetsos, *Spinors and the AdS/CFT correspondence*, *Phys. Lett.* **B 431** (1998) 63 [hep-th/9803251].

[27] A. Volovich, *Rarita-schwinger field in the AdS/CFT correspondence*, *J. High Energy Phys.* **09** (1998) 022 [hep-th/9809009].

[28] A.S. Koshelev and O.A. Rytchkov, *Note on the massive Rarita-Schwinger field in the AdS/CFT correspondence*, *Phys. Lett.* **B 450** (1999) 368 [hep-th/9812238].

[29] P.K. Townsend, *Positive energy and the scalar potential in higher dimensional (super)gravity theories*, *Phys. Lett.* **B 148** (1984) 55.

[30] K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, *Phys. Lett.* **B 468** (1999) 46 [hep-th/9909070].

[31] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics*, *J. High Energy Phys.* **12** (1998) 022 [hep-th/9810126].

[32] D. Anselmi, L. Girardello, M. Porrati and A. Zaffaroni, *A note on the holographic beta and c functions*, *Phys. Lett.* **B 481** (2000) 346 [hep-th/0002066].

[33] J. Distler and F. Zamora, *Non-supersymmetric conformal field theories from stable anti-de Sitter spaces*, *Adv. Theor. Math. Phys.* **2** (1999) 1405 [hep-th/9810206].

[34] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[35] E. Witten, *On the conformal field theory of the Higgs branch*, *J. High Energy Phys.* **07** (1997) 003 [hep-th/9707093].

[36] D.-E. Diaconescu and N. Seiberg, *The coulomb branch of (4,4) supersymmetric field theories in two dimensions*, *J. High Energy Phys.* **07** (1997) 001 [hep-th/9707158].

[37] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Exact Gell-Mann-Low function of supersymmetric Yang-Mills theories from instanton calculus*, *Nucl. Phys.* **B 229** (1983) 381.

[38] O. DeWolfe and D.Z. Freedman, *Notes on fluctuations and correlation functions in holographic renormalization group flows*, hep-th/0002226.

[39] G. Arutyunov, S. Frolov and S. Theisen, *A note on gravity-scalar fluctuations in holographic RG flow geometries*, *Phys. Lett.* **B 484** (2000) 295 [hep-th/0003116].

[40] V. Balasubramanian and P. Kraus, *A stress tensor for anti-de Sitter gravity*, *Commun. Math. Phys.* **208** (1999) 413 [hep-th/9902121].

[41] R.C. Myers, *Stress tensors and Casimir energies in the AdS/CFT correspondence*, hep-th/9903203.

[42] M. Berg and H. Samtleben, in progress.

[43] M. Bianchi, O. DeWolfe, D.Z. Freedman and K. Pilch, *Anatomy of two holographic renormalization group flows*, *J. High Energy Phys.* **01** (2001) 021 [hep-th/0009156].

[44] W. Muck, *Correlation functions in holographic renormalization group flows*, *Nucl. Phys.* **B 620** (2002) 477 [hep-th/0105270].

[45] A. Brandhuber and K. Sfetsos, *Wilson loops from multicentre and rotating branes, mass gaps and phase structure in gauge theories*, *Adv. Theor. Math. Phys.* **3** (1999) 851 [hep-th/9906201].

[46] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, *J. High Energy Phys.* **07** (2000) 038 [hep-th/9906194].

[47] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[48] D. Martelli and A. Miemiec, *CFT/CFT interpolating RG flows and the holographic c-function*, *J. High Energy Phys.* **04** (2002) 027 [hep-th/0112150].

[49] M. Porrati and A. Starinets, *RG fixed points in supergravity duals of 4-d field theory and asymptotically AdS spaces*, *Phys. Lett.* **B 454** (1999) 77 [hep-th/9903085].

[50] J.F. Morales and M. Trigiante, *Walls from fluxes: an analytic RG-flow*, *J. High Energy Phys.* **02** (2002) 018 [hep-th/0112059].